A Linear Estimator for Joint Synchronization and Localization in Wireless Sensor Networks

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***Abstract*—In this paper, joint sensor synchronization and localization using time-of-arrival measurements is studied. In wireless sensor networks, the accuracy of the clock synchronization among nodes has a great impact on the performance of the localization using time-based ranging methods. The clocks of the anchor nodes are typically synchronized with each other, while those of the source nodes must be synchronized with the anchor nodes. Each source node has its own clock characterized by clock offset and clock skew. Synchronization is the process of determining these clock parameters for the source node, while localization is the process of estimating its location. Generally, the estimation problem is broken down into two subproblems, where the synchronization is first performed and then the source node is localized. However, in this paper, a joint synchronization and localization framework is considered and examined, as it is expected to provide better accuracy, especially in dynamic networks. The system model for joint synchronization and localization is first introduced. The maximum likelihood (ML) estimator is then derived which is shown to be highly nonlinear and nonconvex. The ML estimator does not have a closed-form solution and must be solved by computationally complex and iterative algorithms. A novel linear estimator is derived which has a closed-form solution with significantly lower complexity. The performance of the proposed linear estimator is evaluated through computer simulations. Results show that the proposed linear estimator outperforms the previously considered estimators, especially in low signal-to-noise ratios.**

***Index Terms*—sensor localization and synchronization, maximum likelihood (ML), linear least squares (LLS).**

I. Introduction

Wireless sensor networks (WSNs) have been the subject of much interest during the past decades, mainly because of their wide civilian, commercial, and military applications. Synchronization and localization are two of the main components in WSNs. Node locations in the network are required to make their information meaningful [1]. Localization is typically performed by collecting measurements within the network without any aid of external resources such as the Global Positioning System (GPS) [2], [3]. Synchronization is also required in WSNs, since many operations such as power management, data fusion, spectrum allocation, and especially localization depend on it [4].

Generally speaking, a WSN consists of a series of anchor nodes with known locations whose clocks are also synchronized and a number of source nodes with unknown locations whose clocks must be synchronized. The lack of synchronization among nodes in a WSN is mainly due to their different clock parameters (clock offset and skew). Hence, the problem at hand is not only to synchronize the clocks of the source nodes but also to estimate their locations. Typically in asynchronous networks, the clocks at the source nodes are first synchronized and then localization is performed [5], [6]. However, this approach can lead to poor synchronization performance which dramatically impacts the localization accuracy. Due to a close relationship between synchronization and localization, several studies have focused on joint synchronization and localization where they are performed simultaneously. It has been shown that joint synchronization and localization can provide significant improvements over the two-step approach, especially in terms of localization accuracy [7]–[9].

The ML estimator is a popular estimator which provides the optimal accuracy. However, its cost function is severely nonlinear and nonconvex. Due to the nonlinear nature of the cost function, the ML estimator does not have a closed-form solution and can be solved approximately by iterative methods which require an appropriate initialization [10], [11]. The performance of these iterative methods is highly dependent on their initial point. An iterative method may converge to a local minimum (or a saddle point) resulting in large estimation errors. Convex relaxation techniques [12]–[14] and linear estimators [15]–[17] have been introduced to deal with this problem. In the former, the nonlinear and nonconvex ML problem is relaxed into a convex optimization problem such as semidefinite programming (SDP) or second order cone programming (SOCP) [7]. In the latter, the system model is linearized and a linear least squares (LLS) estimator is applied. The advantage of the linear estimators is that they have closed-form solutions. The downside of these techniques is that they are sub-optimal and good performance cannot be expected in all situations.

In this work, a linear estimator for joint sensor synchronization and localization in WSNs is studied. The system model is first introduced and the corresponding ML estimator is then formulated. A novel linear estimator is derived by linearizing the nonlinear system model to a linear one. Although several linear estimators have been previously introduced for this problem in the literature, the proposed estimator has significantly better accuracy in exchange for small complexity growth. The Cramér-Rao lower bound (CRLB) is used as a benchmark for comparisons. The performance of the proposed estimator is compared with those of the ML and other previously considered estimators through computer simulations.

Notation. The following notation is used throughout the paper. Lowercase and uppercase letters denote scalar values. Bold uppercase and bold lowercase letters denote matrices and vectors, respectively. $\mathbf{I}_M$ and $\mathbf{0}_M$ denote the $M \times M$ identity and the $M \times M$ zero matrices, respectively. $\mathbf{1}_M$ represents the $M \times 1$ vector of all ones. $\|\cdot\|$ denotes the ℓ_2 norm and the operator $\odot$ denotes the element-wise product. $[\mathbf{a}]_i$ and $[\mathbf{A}]_{i,j}$ denote the ith element of vector $\mathbf{a}$ and the element at the ith row and jth column of matrix $\mathbf{A}$, respectively.

II. System Model

The internal clocks of the nodes are assumed to be imperfect which causes the internal time drift away from the reference time. The internal time is modeled as a function of the reference as follows [4], [8], [9]

$$t_i = \omega_i t + \theta_i \tag{1}$$

where t_i and t are the internal time of the ith node and the reference time, respectively. ω_i is the *clock skew* accounting for the different rates of change in the times at different clocks and θ_i represents the *clock offset* accounting for the difference in clock readings between two clocks at the same moment in time. Each node has a unique clock meaning that the clock skew and offset are different for each node.

Consider a network with M anchor nodes and one source node. Denote by $\mathbf{x} \in \mathbb{R}^2$, ω_x, and θ_x the unknown coordinates, the clock skew, and the clock offset of the source node, respectively. Denote by $\mathbf{y}_i \in \mathbb{R}^2$, ω_i, and θ_i the known coordinates, the clock skew, and the clock offset of the ith anchor node, respectively. The synchronization and localization are performed simultaneously meaning that the clock parameters and location of the source nodes are jointly estimated from a series of noisy timing synchronization measurements collected within the network. There are two common clock synchronization techniques in WSNs: one-way message dissemination and two-way message exchanges. In the former, either the source node (forward link) or the anchor node (backward link) transmits the synchronization messages [4]. In the latter, both source and anchor nodes transmit synchronization messages as shown in Fig. 1. Several rounds of messages are typically exchanged between the nodes to achieve higher accuracy. At the mth round of message exchange, the source node transmits the forward signal at the time stamp T_{im} and the anchor node i receives the signal at the time stamp R_{im}. The anchor node i then sends back a signal at $\bar{T}_{im}$ and the source node captures the backward signal at $\bar{R}_{im}$. The time stamps T_{im} and $\bar{R}_{im}$ are reported based on the internal clock of the source node, while R_{im} and $\bar{T}_{im}$ are reported based on the internal clock of anchor node i. The measured time stamps at the receivers are modeled as [7], [8]

$$\begin{aligned} R_{im} &= \frac{\omega_i}{\omega_x} T_{im} + \omega_i(t_i + n_{im}) - \frac{\omega_i}{\omega_x}\theta_x + \theta_i \\ \bar{R}_{im} &= \frac{\omega_x}{\omega_i} \bar{T}_{im} + \omega_x(t_i + \bar{n}_{im}) - \frac{\omega_x}{\omega_i}\theta_i + \theta_x \end{aligned} \tag{2}$$

where t_i is the propagation time between the source node and

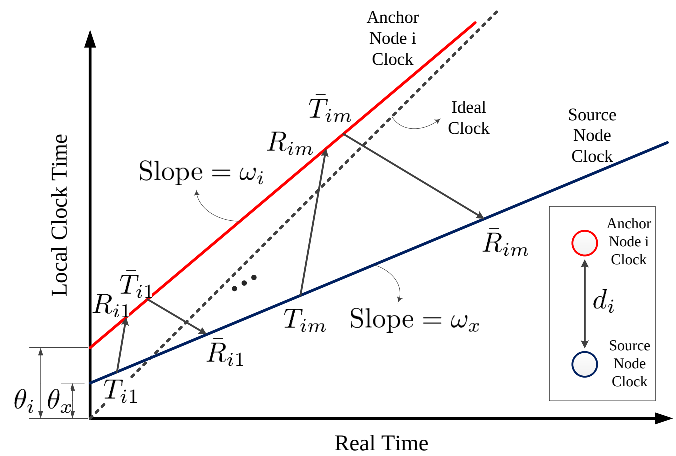


Fig. 1. The two-way message exchange between the source node and anchor node with different clock skews and clock offsets.

the ith anchor node

$$\begin{aligned} t_i &= \frac{1}{c} d_i \\ &= \frac{1}{c}\|\mathbf{x} - \mathbf{y}_i\| \\ &= \frac{1}{c}\sqrt{(\mathbf{x} - \mathbf{y}_i)^{\mathrm{T}}(\mathbf{x} - \mathbf{y}_i)} \end{aligned} \tag{3}$$

where c is the propagation velocity. The terms n_{im} and $\bar{n}_{im}$ represent measurement errors which are modeled as independent and identically distributed (i.i.d.) Gaussian random variables with variance σ_i^2 and $\bar{\sigma}_i^2$, respectively. The variances of the measurements do not vary with time. However, they are dependent on the received SNR and typically modeled as [18]

$$\sigma_i^2 = \mu_i d_i^{\gamma}, \quad \bar{\sigma}_i^2 = \mu_x d_i^{\gamma} \tag{4}$$

where μ_i and μ_x define the relationship between the noise variance and the true distance and their values are dependent on the propagation environment and hardware implementations. γ is the path-loss exponent which typically varies between 2 (free space) and 4 (harsh environments) [19], [20]. Note that all four time stamps $\{T_{im}, R_{im}, \bar{T}_{im}, \bar{R}_{im}\}$ are available to the anchor nodes which are used to estimate the location and clock parameters of the source node.

We assume that during L rounds of message exchanges, the clock parameters and location of the source node do not change. To further simplify (2), the L rounds of message exchanges can be averaged as

$$\begin{aligned} R_i &= \frac{\omega_i}{\omega_x} T_i + \omega_i(t_i + n_i) - \frac{\omega_i}{\omega_x}\theta_x + \theta_i \\ \bar{R}_i &= \frac{\omega_x}{\omega_i} \bar{T}_i + \omega_x(t_i + \bar{n}_i) - \frac{\omega_x}{\omega_i}\theta_i + \theta_x \end{aligned} \tag{5}$$

where

$$\begin{aligned} T_i &= \frac{1}{L}\sum_{m=1}^{L} T_{im}, \quad R_i = \frac{1}{L}\sum_{m=1}^{L} R_{im} \\ \bar{T}_i &= \frac{1}{L}\sum_{m=1}^{L} \bar{T}_{im}, \quad \bar{R}_i = \frac{1}{L}\sum_{m=1}^{L} \bar{R}_{im} \end{aligned} \tag{6}$$

and n_i and $\bar{n}_i$ are i.i.d. Gaussian random variables with variance σ_i^2/L and $\bar{\sigma}_i^2/L$, respectively. It can be seen from (5) that collecting more rounds of measurements decreases the effect of the measurement noise which leads to more accurate localization and synchronization performance.

III. Maximum Likelihood Estimator

Because of several attractive properties, the ML estimator is a very popular estimator. An important property of the ML estimator is that it is asymptotically optimal and it can reach the CRLB when the number of measurements tends to infinity [21, Ch. 7]. In other words, there is no unbiased estimator that performs better than the ML estimator. Note that the CRLB expresses a lower bound on the variance of any unbiased estimator [21, Ch. 3]. The CRLB of the proposed system model is derived in [7]. The ML estimator is obtained by maximizing the likelihood function over the unknown parameters. To make the expressions simpler, two new auxiliary variables are introduced and the model in (5) is rewritten as

$$\begin{aligned} n_i &= R_i\beta_i - d_i - T_i\beta_x + \alpha_x - \alpha_i \\ \bar{n}_i &= \bar{R}_i\beta_x - d_i - \bar{T}_i\beta_i - \alpha_x + \alpha_i \end{aligned} \tag{7}$$

where $\beta_i = c/\omega_i$, $\alpha_i = c\theta_i/\omega_i$, $\beta_x = c/\omega_x$, and $\alpha_x = c\theta_x/\omega_x$. Note that in (7), the standard deviation of measurement noises is expressed in meters rather than seconds. Let $\boldsymbol{\varphi} = [\mathbf{x}^{\mathrm{T}}, \beta_x, \alpha_x]^{\mathrm{T}}$ be the vector of the unknown parameters to be estimated. Since n_i and $\bar{n}_i$ in (7) are zero-mean Gaussian random variables, the ML estimator of $\boldsymbol{\varphi}$ is simply obtained by the following minimization problem

$$\begin{aligned} \hat{\boldsymbol{\varphi}} = \underset{\boldsymbol{\varphi}\in\mathbb{R}^4}{\arg\min} \; & \sum_{i=1}^{M} \sigma_i^{-2} \left(R_i\beta_i - d_i - T_i\beta_x + \alpha_x - \alpha_i\right)^2 \\ & + \sum_{i=1}^{M} \bar{\sigma}_i^{-2} \left(\bar{R}_i\beta_x - d_i - \bar{T}_i\beta_i - \alpha_x + \alpha_i\right)^2 . \end{aligned} \tag{8}$$

Now by using the invariance property, the estimates of the clock parameters are obtained by $\hat{\omega}_x = c/\hat{\beta}_x$ and $\hat{\theta}_x = \hat{\alpha}_x/\hat{\beta}_x$. As mentioned earlier, the main problem of the ML estimator is that its cost function is highly nonlinear and nonconvex. Having no closed-form solution, it must be solved approximately by iterative numerical techniques [21], [22]. As the cost function is nonconvex, there is no guarantee that the solver converges to the global minimum even when the initial point is close to the solution. The difficulty in finding the solution of the ML estimator leads us to employ suboptimal estimators, such as SDP and linear estimators. An SDP estimator for this problem is derived in [7]. A novel linear estimator is introduced in the next section.

IV. Linear Least Squares Estimator

In this section, the proposed linear estimator is derived. Unlike the ML estimator, the proposed linear estimator has a closed-from solution. Moreover, it has significantly lower computational complexity than the ML estimator. To obtain the proposed linear estimator, the system model needs to first linearized in terms of the unknown variables. The optimization problem is then formulated as a linear least squares (LLS) problem which has a closed-form solution. Rearranging (7) yields

$$\begin{aligned} \xi_i - T_i\beta_x &= d_i - \alpha_x + n_i \\ \bar{\xi}_i + \bar{R}_i\beta_x &= d_i + \alpha_x + \bar{n}_i \end{aligned} \tag{9}$$

where $\xi_i = R_i\beta_i - \alpha_i$ and $\bar{\xi}_i = \alpha_i - \bar{T}_i\beta_i$. To eliminate the dependency of (9) on the unknown variable α_x, an anchor node is selected as a reference and its measurements are subtracted from all other measurements. Let $r \in \{1, \ldots, M\}$ be the index of the reference anchor node and $\mathbf{y}_r$ be its location. Therefore, (9) is expressed as

$$\begin{aligned} \xi_{ir} - T_{ir}\beta_x + d_r &= d_i + n_{ir} \\ \bar{\xi}_{ir} + \bar{R}_{ir}\beta_x + d_r &= d_i + \bar{n}_{ir} \end{aligned} \tag{10}$$

where

$$\begin{aligned} T_{ir} &= T_i - T_r, & \bar{T}_{ir} &= \bar{T}_i - \bar{T}_r \\ R_{ir} &= R_i - R_r, & \bar{R}_{ir} &= \bar{R}_i - \bar{R}_r \\ \xi_{ir} &= \xi_i - \xi_r, & \bar{\xi}_{ir} &= \bar{\xi}_i - \bar{\xi}_r \\ n_{ir} &= n_i - n_r, & \bar{n}_{ir} &= \bar{n}_i - \bar{n}_r. \end{aligned}$$

Let us define two auxiliary variables as

$$\begin{aligned} \mathbf{y}_{ir} &= \mathbf{y}_i - \mathbf{y}_r \\ \mathbf{x}_r &= \mathbf{x} - \mathbf{y}_r. \end{aligned} \tag{11}$$

Then, we can write d_i and d_r as

$$\begin{aligned} d_i &= \|\mathbf{x} - \mathbf{y}_i\| \\ &= \|(\mathbf{x} - \mathbf{y}_r) - (\mathbf{y}_i - \mathbf{y}_r)\| \\ &= \|\mathbf{x}_r - \mathbf{y}_{ir}\| \\ d_r &= \|\mathbf{x} - \mathbf{y}_r\| \\ &= \|\mathbf{x}_r\|. \end{aligned} \tag{12}$$

To progress, an approximation must be applied to the model. The clock skew of the source node can be expressed as $\omega_x = 1 + \delta$, where $\delta \ll 1$ is a relatively small value. For sufficiently small δ, we have [16]

$$\beta_x = \frac{1}{\omega_x} = 1 - \delta. \tag{13}$$

Applying the above approximation and squaring both sides, (10) can be written as an overdetermined linear system as

$$\begin{aligned} \mathbf{g}_{i,1}^{\mathrm{T}}\boldsymbol{\vartheta}_1 &= h_{i,1} + \epsilon_{i,1} \\ \bar{\mathbf{g}}_{i,1}^{\mathrm{T}}\boldsymbol{\vartheta}_1 &= \bar{h}_{i,1} + \bar{\epsilon}_{i,1} \end{aligned} \tag{14}$$

where $\boldsymbol{\vartheta}_1 = [\mathbf{x}_r^{\mathrm{T}}, \|\mathbf{x}_r\|, \delta]^{\mathrm{T}}$, and

$$\begin{aligned} \mathbf{g}_{i,1} &= \begin{bmatrix} -2\mathbf{y}_{ir}^{\mathrm{T}} & -2\xi_{ir} + 2T_{ir} & 2T_{ir}^2 + 2T_{ir}\xi_{ir} \end{bmatrix} \\ \bar{\mathbf{g}}_{i,1} &= \begin{bmatrix} -2\mathbf{y}_{ir}^{\mathrm{T}} & -2\bar{R}_{ir} + 2\bar{\xi}_{ir} & 2\bar{R}_{ir}^2 + 2\bar{R}_{ir}\bar{\xi}_{ir} \end{bmatrix} \\ h_{i,1} &= \xi_{ir}^2 + T_{ir}^2 - \mathbf{y}_{ir}^{\mathrm{T}}\mathbf{y}_{ir} - 2T_{ir}\xi_{ir} \\ \bar{h}_{i,1} &= \bar{\xi}_{ir}^2 + \bar{R}_{ir}^2 - \mathbf{y}_{ir}^{\mathrm{T}}\mathbf{y}_{ir} - 2\bar{R}_{ir}\bar{\xi}_{ir} \end{aligned}$$

$$\epsilon_{i,1} = 2d_i n_{ir} - 2T_{ir}\delta\|\mathbf{x}_r\|$$
$$\bar{\epsilon}_{i,1} = 2d_i \bar{n}_{ir} + 2\bar{R}_{ir}\delta\|\mathbf{x}_r\|.$$

Writing (14) for $\{i|i = 1, \ldots, M, i \neq r\}$ in matrix form yields

$$\mathbf{G}_1\boldsymbol{\vartheta}_1 = \mathbf{h}_1 + \boldsymbol{\epsilon}_1. \quad (15)$$

The unknown vector, $\boldsymbol{\vartheta}_1$, can be estimated by a least squares estimator. Since the above model is a linear function of the unknown vector, the least squares estimator has a closed-form solution as [21]

$$\hat{\boldsymbol{\vartheta}}_1 = \left(\mathbf{G}_1^{\mathrm{T}}\mathbf{C}_1^{-1}\mathbf{G}_1\right)^{-1}\mathbf{G}_1^{\mathrm{T}}\mathbf{C}_1^{-1}\left(\mathbf{h}_1 - \mathrm{E}\{\boldsymbol{\epsilon}_1\}\right) \quad (16)$$

where $\mathbf{C}_1 = \mathrm{E}\{\boldsymbol{\epsilon}_1\boldsymbol{\epsilon}_1^{\mathrm{T}}\}$. The mean and covariance of the error are obtained by

$$\begin{aligned}
\mathrm{E}\{\epsilon_{i,1}\} &= -2T_{ir}\delta\|\mathbf{x}_r\| \\
\mathrm{E}\{\bar{\epsilon}_{i,1}\} &= 2\delta\|\mathbf{x}_r\|\omega_x(d_{ir} - \bar{\xi}_{ir}) \\
\mathrm{E}\{\epsilon_{i,1}^2\} &= 4d_i^2\sigma_{ir}^2 + 4T_{ir}^2\delta^2\|\mathbf{x}_r\|^2 \\
\mathrm{E}\{\bar{\epsilon}_{i,1}^2\} &= 4d_i^2\bar{\sigma}_{ir}^2 + 4\bar{R}_{ir}^2\delta^2\|\mathbf{x}_r\|^2 \\
\mathrm{E}\{\epsilon_{i,1}\epsilon_{k,1}\} &= 4d_i d_k\sigma_r^2 \\
\mathrm{E}\{\bar{\epsilon}_{i,1}\bar{\epsilon}_{k,1}\} &= 4(d_i + \omega_x\delta\|\mathbf{x}_r\|)(d_k + \omega_x\delta\|\mathbf{x}_r\|)\bar{\sigma}_r^2.
\end{aligned}$$

Note than the mean and covariance of the error are dependent on the true values of $\mathbf{x}_r$ and δ which are unknown. To deal with this problem, the solution of (16) is first obtained by setting $\mathbf{C}_1 = \mathbf{I}$ and $\mathrm{E}\{\boldsymbol{\epsilon}_1\} = \mathbf{0}$. The mean and covariance matrix are then calculated from the estimates of $\mathbf{x}_r$ and δ, and again the solution of (16) is calculated by the approximate mean and covariance matrix. In order to achieve a more accurate estimate, the relationship between the elements of the vector $\hat{\boldsymbol{\vartheta}}_1$ is exploited. To do that, the following set of linear equations can be formed

$$\mathbf{G}_2\boldsymbol{\vartheta}_2 = \mathbf{h}_2 + \boldsymbol{\epsilon}_2 \quad (17)$$

where

$$\begin{aligned}
\boldsymbol{\vartheta}_2 &= \mathbf{x}_r \odot \mathbf{x}_r \\
\mathbf{h}_2 &= [\boldsymbol{\vartheta}_2]_{1:3} \odot [\boldsymbol{\vartheta}_2]_{1:3} \\
\mathbf{G}_2 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}.
\end{aligned}$$

The least squares solution of (17) is calculated as

$$\hat{\boldsymbol{\vartheta}}_2 = \left(\mathbf{G}_2^{\mathrm{T}}\mathbf{C}_2^{-1}\mathbf{G}_2\right)^{-1}\mathbf{G}_2^{\mathrm{T}}\mathbf{C}_2^{-1}\mathbf{h}_2 \quad (18)$$

where $\mathbf{C}_2 = \mathbf{D}[\mathbf{C}_\vartheta]_{1:3,1:3}\mathbf{D}$, $\mathbf{C}_\vartheta = \left(\mathbf{G}_1^{\mathrm{T}}\mathbf{C}_1^{-1}\mathbf{G}_1\right)^{-1}$, and $\mathbf{D} = \mathrm{diag}\{[\hat{\boldsymbol{\vartheta}}_1]_{1:3}\}$. Eventually, the estimate of the source location is computed as

$$\hat{\mathbf{x}} = \mathrm{sgn}([\hat{\boldsymbol{\vartheta}}_1]_{1:2}) \odot \sqrt{|\hat{\boldsymbol{\vartheta}}_2|} + \mathbf{y}_r \quad (19)$$

where $\mathrm{sgn}(\cdot)$ denotes the element-wise sign function. Note that in (17), a refinement is applied to the source location $\mathbf{x}$ but not to δ. Now, the goal is to use the refined estimate of $\mathbf{x}$ to improve the estimation accuracy of δ. To do that, (10) can be written as

$$\begin{aligned}
T_{ir}\beta_x &= \hat{d}_{ir} - \xi_{ir} + n_{ir} \\
\bar{R}_{ir}\beta_x &= \hat{d}_{ir} - \bar{\xi}_{ir} + \bar{n}_{ir}
\end{aligned} \quad (20)$$

where

$$\hat{d}_{ir} = \sqrt{(\hat{\mathbf{x}}_r - \mathbf{y}_{ir})^{\mathrm{T}}(\hat{\mathbf{x}}_r - \mathbf{y}_{ir})}$$

and $\hat{\mathbf{x}}_r = \hat{\mathbf{x}} - \mathbf{y}_r$. The ML estimator can be applied to (20) to estimate β_x from (20). Since the model in (20) is linear in terms of the unknown variable β_x, its ML estimator can be simply calculated by

$$\hat{\beta}_x = \left(\mathbf{g}_3^{\mathrm{T}}\mathbf{C}_3^{-1}\mathbf{g}_3\right)^{-1}\mathbf{g}_3^{\mathrm{T}}\mathbf{C}_3^{-1}\mathbf{h}_3 \quad (21)$$

where $\mathbf{C}_3 = \mathrm{E}\{\mathbf{n}_r\mathbf{n}_r^{\mathrm{T}}\}$ and

$$\begin{aligned}
g_{i,3} &= T_{ir}, & \bar{g}_{i,3} &= \bar{R}_{ir} \\
h_{i,3} &= \hat{d}_{ir} - \xi_{ir}, & \bar{h}_{i,3} &= \hat{d}_{ir} - \bar{\xi}_{ir}.
\end{aligned}$$

Now we would like to introduce another step to further improve the estimate of the source location. The relationship between the true value of $\mathbf{x}$ and its estimate $\hat{\mathbf{x}}$ in (19) can be expressed as

$$\mathbf{x} = \hat{\mathbf{x}} + \Delta\mathbf{x}. \quad (22)$$

Applying a first-order Taylor series expansion to d_{ir} yields

$$\begin{aligned}
d_{ir} &= \|\mathbf{x} - \mathbf{y}_i\| - \|\mathbf{x} - \mathbf{y}_r\| \\
&= \|(\hat{\mathbf{x}} + \Delta\mathbf{x}) - \mathbf{y}_i\| - \|(\hat{\mathbf{x}} + \Delta\mathbf{x}) - \mathbf{y}_r\| \\
&= \hat{d}_i - \hat{d}_r + (\hat{\mathbf{x}} - \mathbf{y}_i)^{\mathrm{T}}\Delta\mathbf{x}/\hat{d}_i - (\hat{\mathbf{x}} - \mathbf{y}_r)^{\mathrm{T}}\Delta\mathbf{x}/\hat{d}_r \\
&= \hat{d}_{ir} + [(\hat{\mathbf{x}} - \mathbf{y}_i)/\hat{d}_i - (\hat{\mathbf{x}} - \mathbf{y}_r)/\hat{d}_r]^{\mathrm{T}}\Delta\mathbf{x}.
\end{aligned} \quad (23)$$

Plugging (23) in (10) yields

$$\begin{aligned}
\xi_{ir} - T_{ir}\hat{\beta}_x - \hat{d}_{ir} &= \mathbf{g}_{i,4}^{\mathrm{T}}\Delta\mathbf{x} + n_{ir} \\
\bar{\xi}_{ir} + \bar{R}_{ir}\hat{\beta}_x - \hat{d}_{ir} &= \bar{\mathbf{g}}_{i,4}^{\mathrm{T}}\Delta\mathbf{x} + \bar{n}_{ir}
\end{aligned} \quad (24)$$

where

$$\begin{aligned}
\mathbf{g}_{i,4} &= \bar{\mathbf{g}}_{i,4} \\
&= (\hat{\mathbf{x}} - \mathbf{y}_i)/\hat{d}_i - (\hat{\mathbf{x}} - \mathbf{y}_r)/\hat{d}_r.
\end{aligned}$$

We can write (24) in matrix form as

$$\mathbf{G}_4\Delta\mathbf{x} = \mathbf{h}_4 + \mathbf{n}_r \quad (25)$$

where the elements of the vector $\mathbf{h}_4$ are given by

$$\begin{aligned}
h_{i,4} &= \xi_{ir} - T_{ir}\hat{\beta}_x - \hat{d}_{ir} \\
\bar{h}_{i,4} &= \bar{\xi}_{ir} + \bar{R}_{ir}\hat{\beta}_x - \hat{d}_{ir}.
\end{aligned} \quad (26)$$

Note that (25) is derived based on the fact that $\Delta\mathbf{x}$ is sufficiently small. Therefore, the solution of (25) can be found by a regularized least squares estimator which is obtained by

$$\Delta\hat{\mathbf{x}} = \underset{\Delta\mathbf{x}\in\mathbb{R}^2}{\arg\min}\ \|\mathbf{G}_4\Delta\mathbf{x} - \mathbf{h}_4\|^2 + \lambda\|\Delta\mathbf{x}\|^2 \quad (27)$$

where λ is a regularization parameter controlling the trade-off between the first and second terms. In (27), we not only minimize the cost function but also keep $\Delta\mathbf{x}$ small depending

TABLE I
THE RUNNING TIME OF THE CONSIDERED ESTIMATORS.

Estimator	Description	Time [ms]
ML	The ML estimator in (8)	216.91
NEW	The proposed estimator in (29)	1.76
SDP	A SDP estimator in [7]	112.30
LLS I	A linear estimator in [8]	0.54
LLS II	A linear estimator in [24]	0.28

on λ. The problem in (27) has a closed-form solution given by [23]

$$\Delta\hat{\mathbf{x}} = \left(\mathbf{G}_4^{\mathrm{T}}\mathbf{C}_3^{-1}\mathbf{G}_4 + \lambda\mathbf{I}\right)^{-1}\mathbf{G}_4^{\mathrm{T}}\mathbf{C}_3^{-1}\mathbf{h}_4. \quad (28)$$

Hence, the final estimate of the source location is obtained by

$$\tilde{\mathbf{x}} = \hat{\mathbf{x}} + \Delta\hat{\mathbf{x}}. \quad (29)$$

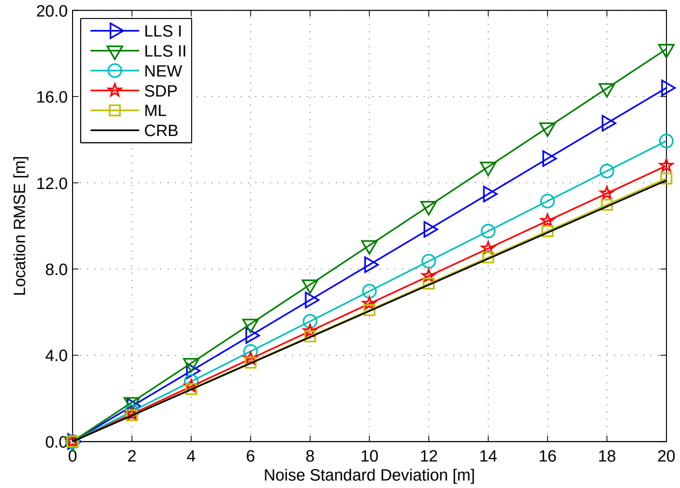
The estimate of α_x can be obtained by plugging (29) and (21) in (9) and applying a LLS estimator. Moreover, the clock parameters (skew and offset) of the source node can be simply determined from $\hat{\beta}_x$ and $\hat{\alpha}_x$, as shown in (8).

V. SIMULATION RESULTS

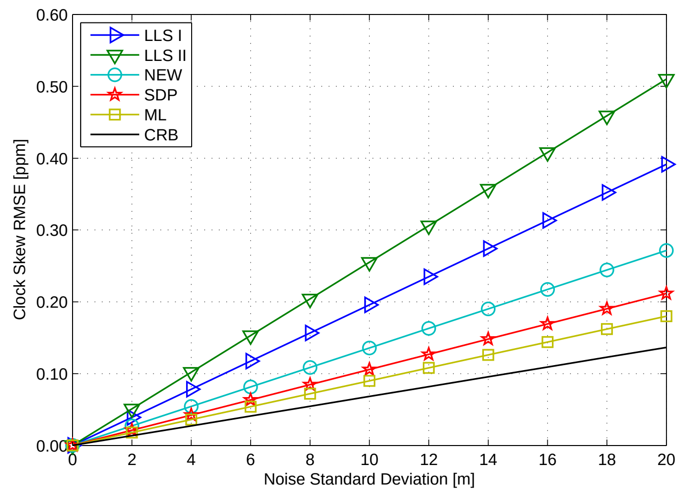
Computer simulations are conducted to evaluate the performance of the proposed linear estimator. The ML estimator is solved by the MATLAB routine `fminunc` and its solver is initialized with the true values in an attempt to avoid possible local minima and to represent a best-case performance. The regularization parameter, λ, of the proposed estimator in (29) is set to 0.1. Besides the ML estimator, three previously considered estimators are selected for comparisons: a SDP estimator derived in [7] and two linear estimators given in [8] and [24]. A summary of the considered estimators and their labels are provided in Table I. Note that the proposed linear estimator is labeled as *NEW*.

A network with four anchor nodes and a source nodes is considered. The anchor nodes are located at: $[10, 10]^{\mathrm{T}}$, $[10, 90]^{\mathrm{T}}$, $[90, 10]^{\mathrm{T}}$, and $[90, 90]^{\mathrm{T}}$ m. The source node is randomly placed in a region of 100m $\times$ 100m. The clock offsets and skews of the nodes are drawn from a uniform distribution $\mathcal{U}(0, 10^{-1})$ µs and $\mathcal{U}(0.95, 1.05)$, respectively. The transmission times of the nodes T_{im} and $\bar{T}_{im}$ are drawn from a uniform distribution $\mathcal{U}(5m, 5m+1)$ s and $\mathcal{U}(5m+3, 5m+4)$ s, respectively. The variances of the measurement noises are determined based on (4). The value of the path-loss exponent γ is set to 2 and assumed to be the same for all nodes. The values of μ_x and μ_i are also assumed to be the same for all nodes. For each location of the source node, 100 realizations are performed where the clock parameters, transmission times, and noises are randomized. It is assumed that two rounds of measurements, $L = 2$, are collocated among nodes.

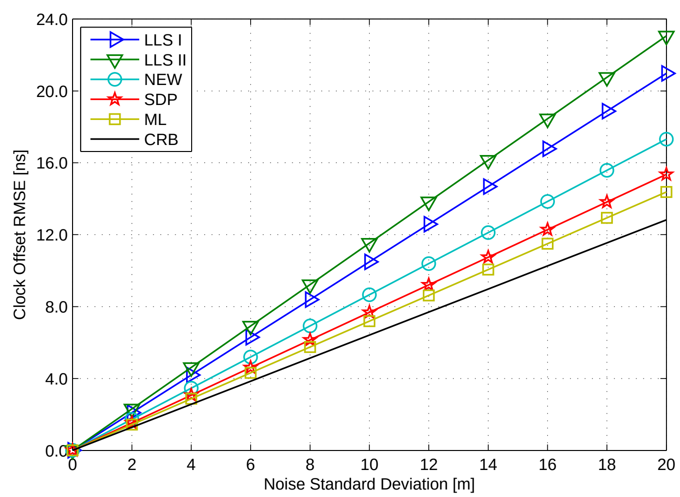
Fig. 2 shows the root-mean-square error (RMSE) performance of the considered estimators versus average standard deviation of the ranging noises. The corresponding running times of the considered estimators are provided in Table I. Note that the plotted performances are obtained by averaging



(a) Location



(b) Clock Skew



(c) Clock Offset

Fig. 2. The RMSE performance of the considered estimators for the location, clock skew, clock offset of the source node. The proposed linear estimator (labeled as NEW) outperforms the other previously proposed linear estimators. Moreover, the close relationship between the synchronization and localization performance of the estimators can be observed. An estimator which provides better accuracy in synchronization has better performance in localization too.

over all source node locations and noise realizations. Fig. 2a shows that the ML estimator provides the optimal accuracy and its performance is close to the CRLB, especially for low standard deviations of the noise. For higher standard deviations, the ML estimator starts to deviate from the CRLB. The reason is that the ML estimator is asymptotically efficient and it is expected to provide the optimal accuracy only when the number of measurements tends to infinity or the measurement noise is very low. The SDP estimator also performs as well as the ML estimator. Although both ML and SDP estimators perform very well, their running times are significantly higher than those of the linear estimators, as shown in Table I. The proposed linear estimator significantly outperforms other linear estimators (LLS I and LLS II), although its running time is slightly higher. The reason is that the proposed estimator is a five-step estimator, while LLS I and LLS II are a two-step and one-step estimator, respectively. The higher the number of steps, the higher the complexity and the running time are.

There are two factors that make the proposed linear estimator different from the other linear estimators previously proposed in the literature. First, unlike in the previous estimators, in the proposed estimator, the dependency of the clock offset is removed from the model by using differential measurements. In this case, the linearization should be performed over 3 parameters rather than 4 parameters. Second, the proposed estimator exploits the fact that the clock skew of the source node varies around 1 which is a valid assumption in practice. However, such an assumption was not made in the previous estimators. In Fig. 2b and Fig. 2c, the synchronization performance of the considered estimators is compared. The order of estimators remains unchanged in comparison with Fig. 2a. The direct relationship between the synchronization and localization accuracy can be clearly seen. An estimator which provides better accuracy in synchronization has better performance in localization too.

VI. Conclusion

In this paper, the problem of joint synchronization and localization in wireless sensor networks was examined. We assumed that anchor nodes with known locations are perfectly synchronized, while the source node with unknown locations are not synchronized. The system model and its corresponding maximum likelihood (ML) estimator were derived. It was shown that the ML estimator is highly nonlinear which requires intensive computations to obtain its approximate solution. Avoiding the problems associated with the ML estimator, a linear estimator was derived which estimates jointly the location and the clock parameters of the source node. Computer simulations were used to evaluate the performance of the proposed estimator. Simulation results showed that the proposed linear estimator significantly outperforms the previously linear estimators considered in the literature. Having considerably lower running time, the proposed linear estimator performs as nearly well as the ML estimator, especially for low standard deviations of the measurement noise.

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