On the Complexity-Performance Trade-off in Code-Aided Frame Synchronization

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***Abstract*—Next generation wireless communication systems are pushing the limits of both energy efficiency and spectral efficiency. This presents a challenge to other functions in the receiver such as frame synchronization. In this paper we examine the trade-off between increased complexity and the improvement in energy and spectral efficiency of code-aided frame synchronization. Parallel and serial approaches to the frame synchronization problem are considered as well as methods for minimizing their complexity. We identify regions over which code-aided frame synchronization improves performance, with respect to a conventional receiver, while maintaining reasonable complexity.**

I. INTRODUCTION

Next generation wireless communication systems are pushing the limits of both energy efficiency and spectral efficiency. Energy efficiency is aided by advanced error correction codes while spectral efficiency is aided by higher order modulations, MIMO transmission, and the reduction of training overhead. However, this presents a challenge to other functions in the receiver such as frame synchronization. Iterative synchronization techniques allow us to take advantage of advanced error correction codes while minimizing the need for training. The downside of these techniques is the increase in the receiver's complexity. Thus, in this paper we examine the trade-off between increased complexity and the improvement in energy and spectral efficiency.

Frame synchronization is important due to the fact that an incorrect estimate of the frame start time is catastrophic to the detection of the data. Unlike other channel impairments, non-data-aided methods generally do not provide any useful information about the frame start time. Conventional systems add a sync word as a preamble to the frame and rely on correlation-based synchronization. However, the correlation rule is known to be sub-optimal. Maximum likelihood frame synchronization for uncoded data is derived in [1] for BPSK and in [2] for higher-order modulations.

In the literature, a number of frame synchronization algorithms for coded data have been proposed including methods based on the parity check structure of codes [3], [4], on the expectation maximization (EM) algorithm [5], [6], [7], and on minimizing free energy [8]. A survey of this work establishes that reliable frame synchronization can be performed at the low signal-to-noise ratio (SNR) of interest by harnessing the strength of the error correction code. Yet, in much of the present work, this comes at the cost of decoding all possible frame offsets.

List synchronization describes an approach in which a low complexity decision rule is used to create a list of the most likely frame offsets. The list is then processed by a more complex synchronization technique [9], [10]. This approach reduces the complexity associated with implementing a coded-aided technique. List synchronization was explored by Cassaro and Georghiades in [10] where the optimal uncoded decision rule is used for generating a list of high probability offsets (typically four offsets are chosen). The decision rule for the high probability list is based on mode separation in the log likelihood ratios obtained from soft decoding.

In our previous work, we have proposed a Bayesian technique to select, on a frame-by-frame basis, the minimum number of frame offsets which must be processed with a code-aided frame synchronization algorithm [11]. The proposed technique makes use of the highest posterior density (HPD) region of the frame offset and is applied to frame synchronization of burst transmissions without a sync word. In the present work, we apply the method proposed in [11] to a continuous transmission in which a sync word is available to the receiver. A key contribution in this regard is to explore the trade-off between the improvement in performance and the complexity increase with respect to conventional receivers. We identify regions over which code-aided frame synchronization provides good improvement over conventional techniques while maintaining a reasonable complexity.

A limitation of the proposed method, as well as the other frame synchronizers proposed in literature, is that the frame offset estimation is a parallel process. That is, the approach involves maximization of a statistic over all possible frame offsets. The methods are inherently complex because all offsets must be considered even if by a simpler preprocessing stage. Although the HPD region contains the minimum number of offsets a code-aided method must consider, in a particular realization the HPD region may still be large.

Another, simpler approach is to construct the frame synchronization problem as a series of binary hypothesis tests. Given a particular offset, we desire to make a reliable statement as to whether the frame is synchronized (hypothesis $\mathcal{H}_1$) or not synchronized (hypothesis $\mathcal{H}_0$). In other words, we are given the choice that the frame begins at the present offset versus the frame begins at some other offset. The serial approach is common in spread spectrum systems [12] and was used by Mengali *et al.* in [13] for the purpose of node synchronization of convolutional codes. In this paper, we develop a serial method of frame synchronization within the binary hypothesis testing framework. The detector makes use of the sync word as well as extrinsic information from the decoder to make a reliable decision.

II. Frame Synchronization: Parallel Approach

A. System model

Consider the reception of a frame consisting of a known sync word followed by a coded data sequence at an unknown frame offset η. Let the sync word be denoted by $\mathbf{s} = [s_0, \ldots, s_{L-1}]^{\mathrm{T}}$ with length L and the block of coded data symbols be denoted by $\mathbf{a} = [a_0, \ldots, a_{K-1}]^{\mathrm{T}}$ with length K. We consider a continuous transmission of frames where the frame of interest is given by $\mathbf{x} = [\mathbf{s}^{\mathrm{T}}\mathbf{a}^{\mathrm{T}}]^{\mathrm{T}}$ and has length $N = L + K$. The frame offset is denoted by η and may take on values in the set $\eta \in \{0, 1, \ldots, N-1\}$. The symbols are drawn from an Mary digital phase-amplitude modulation with constellation space given by the set $\mathcal{X}$ and unit average power.

B. Conventional estimation

In conventional frame synchronization, the error correction code is ignored by the receiver and the data symbols are modeled as iid random variables drawn with equal probability from the constellation space. The receiver captures $2N-1$ samples in order to obtain a full frame at each possible frame offset. Let $\mathbf{x}^m$ denote the mth frame and x_i^m the ith symbol in the mth frame. The received signal is given by

$$\mathbf{y} = [x_{N-\eta}^{m-1}, \ldots, x_{N-1}^{m-1}, \mathbf{x}^m, x_0^{m+1}, \ldots, x_{N-\eta-2}^{m+1}]^{\mathrm{T}} + \mathbf{n}.$$

where $\mathbf{n}$ is a vector of independent identically distributed (iid) complex Gaussian random variables with variance equal to the noise power spectral density N_0.

Maximum *a posteriori* probability (MAP) estimation of the sync word seeks to maximize the posterior probability of the frame offset $p(\eta|\mathbf{y}) = f(\mathbf{y}|\eta)p(\eta)/f(\mathbf{y})$ where the notation $f(\cdot)$ represents the probability distribution function or likelihood function and $p(\cdot)$ represents the probability mass function. *A priori* all frame offsets are equally likely (i.e., $p(\eta) = 1/N$) and the posterior probabilities are proportional to the likelihood function $f(\mathbf{y}|\eta)$. The likelihood function is given by

$$\begin{aligned} f(\mathbf{y}|\eta) =& (\pi N_0)^{-N/2} \prod_{i=0}^{L-1} \exp\left\{-\frac{1}{N_0}|y_{i+\eta} - s_i|^2\right\} \\ &\cdot \prod_{i=0}^{\eta-1} \sum_{d_i \in \mathcal{X}} \exp\left\{-\frac{1}{N_0}|y_i - d_i|^2\right\} \frac{1}{M} \\ &\cdot \prod_{i=L+\eta}^{2N-2} \sum_{d_i \in \mathcal{X}} \exp\left\{-\frac{1}{N_0}|y_i - d_i|^2\right\} \frac{1}{M}. \end{aligned} \quad (1)$$

Removing terms which are independent of η and dividing by $\prod_{i=0}^{2N-2} \sum_{d_i \in \mathcal{X}} \exp\left\{-\frac{1}{N_0}|y_i - d_i|^2\right\}$, the likelihood function is expressed as given by [2]

$$f(\mathbf{y}|\eta) \propto \frac{\prod_{i=0}^{L-1} \exp\left\{\frac{2}{N_0}\Re\left[y_{i+\eta}s_i^*\right]\right\}}{\prod_{i=0}^{L-1} \sum_{d_i \in \mathcal{X}} \exp\left\{\frac{2}{N_0}\Re\left[y_{i+\eta}d_i^*\right] - \frac{1}{N_0}\left|d_i\right|^2\right\}}.$$

When the modulation is BPSK, the likelihood function is further simplified to [1]

$$f(\mathbf{y}|\eta) \propto \frac{\exp\left\{\frac{2}{N_0}\sum_{i=0}^{L-1} y_{i+\eta}s_i\right\}}{\prod_{i=0}^{L-1} \cosh\left(\frac{2}{N_0}y_{i+\eta}\right)}.$$

C. HPD region

From the likelihoods derived for the conventional approach, the posterior distribution of the frame offset can be obtained. That is, the posterior distribution is given by

$$p(\eta|\mathbf{y}) = \frac{f(\mathbf{y}|\eta)}{\sum_{i=0}^{N-1} f(\mathbf{y}|\eta = i)}.$$

Given a set $\mathcal{S}$ which contains a subset of the frame offsets, we denote by P_{ex} the probability that the true offset has been excluded from this subset (i.e., $\eta \notin \mathcal{S}$). We desire to find the smallest subset of frame offsets which has a P_{ex} less than or equal to a specified value. This set is known as the highest posterior density (HPD) region which we will denote by $\mathcal{S}$ with size $|\mathcal{S}|$. For example, to limit the probability of exclusion to $P_{ex} = 0.01$, the set of frame offsets $\mathcal{S}$ which must be processed by the iterative receiver is given by the 0.99 HPD region. Mathematically, we can express the $(1 - P_{ex})$ HPD as the smallest set $\mathcal{S}$ such that

$$\sum_{i \in \mathcal{S}} p(\eta = i|\mathbf{y}) \geq 1 - P_{ex}.$$

The frame offsets contained in the HPD region are passed to the code-aided synchronizer in order to make a reliable decision. Using the HPD region allows the receiver to choose on a frame by frame basis how many and which frame offsets to process with the code-aided synchronizer. In this way the complexity of the frame estimation performed by the coded-aided synchronizer is minimized for a specified P_{ex}. Note that the frame error rate (FER) of the receiver will be lower-bound by the P_{ex}.

D. Code-aided frame offset estimation

The code-aided method is based on the EM algorithm [14] which provides a means of iteratively estimating parameter $\boldsymbol{\theta}$ from incomplete data $\mathbf{y}$ when there exists a set of unobserved or missing data $\mathbf{x}$. Maximization is performed on the complete data $\mathbf{z} = [\mathbf{y}, \mathbf{x}]$ which typically simplifies computation. The EM algorithm is given by the following expectation (E) and maximization (M) steps:

$$\text{E step: } Q(\boldsymbol{\theta}, \hat{\boldsymbol{\theta}}^{(p-1)}) = \int_{\mathbf{z}} f(\mathbf{z}|\mathbf{y}, \hat{\boldsymbol{\theta}}^{(p-1)}) \ln f(\mathbf{z}|\boldsymbol{\theta}) d\mathbf{z} \quad (2)$$

$$\text{M step: } \hat{\boldsymbol{\theta}}^{(p)} = \arg\max_{\boldsymbol{\theta}} Q(\boldsymbol{\theta}, \hat{\boldsymbol{\theta}}^{(p-1)}) \quad (3)$$

where p is the iteration number. In the case of discrete parameters the EM algorithm does not converge well and the modification proposed by Wymeersch *et al.* is to compute (2) for all discrete values [6].

Applying the modified EM algorithm to frame offset estimation, we can formulate the method as follows:

$$\hat{\eta} = \arg\max_{\eta \in \mathcal{S}} Q(\eta) \quad (4)$$

where

$$Q(\eta) = \sum_{\mathbf{x}} p(\mathbf{x}|\mathbf{y}, \eta) \ln f(\mathbf{y}|\mathbf{x}, \eta). \quad (5)$$

In our case, maximization is performed over all frame offsets in the HPD region $\mathcal{S}$. In (5), the log likelihood $\ln f(\mathbf{y}|\mathbf{x},\eta)$ can be factored as given by

$$\ln f(\mathbf{y}|\mathbf{x},\eta) \propto \frac{2}{N_0}\sum_{i=0}^{2N-1}\Re\{y_i^* x_i\} - \frac{1}{2}|x_i|^2. \tag{6}$$

Substituting (6) into (5) and rearranging the summations provides the following

$$Q(\eta) = \frac{2}{N_0}\sum_{i=0}^{2N-1}\left[\Re\left\{y_i^*\sum_{x_i\in\mathcal{X}} x_i p(x_i|\mathbf{y},\eta)\right\} - \frac{1}{2}\sum_{x_i\in\mathcal{X}}|x_i|^2 p(x_i|\mathbf{y},\eta)\right]. \tag{7}$$

We observe that the EM framework has reduced the summation over symbol sequences in (5) to symbol-by-symbol summations. The posterior probabilities must be computed for three cases: (1) sync word symbols for $i=\eta,\dots,L-1+\eta$; (2) coded data symbols for $i=L+\eta,\dots,N-1+\eta$; and (3) out-of-frame symbols for $i=0,\dots,\eta-1,N+\eta,\dots,2N-2$. Because the sync word is known to the receiver, the posterior distributions of these symbols are unit impulses. The posterior distributions for the coded data symbols are obtained through message passing on the factor graph of the demodulator and decoder for each frame offset under consideration. Finally, we do not make any assumptions about the out-of-frame symbols (i.e., the symbols from frames other than the one of interest) except that they share the same modulation as the in-frame symbols. Thus, the posterior probabilities are proportional to the likelihood $p(y_i|x_i)$ which is straightforward to evaluate. One advantage of the EM algorithm over other code-aided techniques is the ability to combine knowledge of the sync word into the detector in a theoretically justified manner.

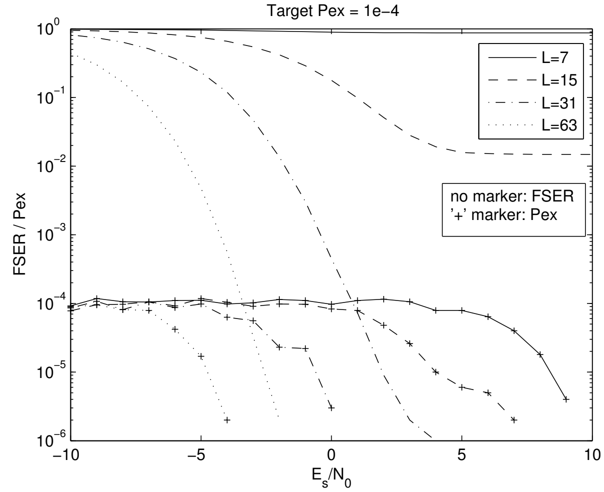

Fig. 1. Performance of conventional synchronization compared to the P_{ex} of the HPD region.

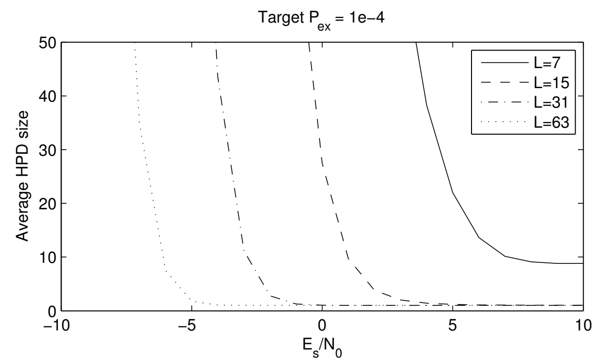

Fig. 2. Average HPD size for a target $P_{ex}=10^{-4}$

E. Numerical results

We begin our analysis of the proposed synchronization method by evaluating the performance of optimal uncoded synchronization and of frame offset selection based on the HPD region. The target probability of exclusion is set to $P_{ex}=10^{-4}$. In this simulation the modulation is BPSK and the size of the data portion of the frame is $K=1000$. For the sync word, *m*-sequences of length L equal to 7, 15, 31, and 63 are chosen. The simulated frame synchronization error rate (FSER) and simulated P_{ex} are shown in Fig. 1.

The simulated P_{ex} begins to roll-off as E_s/N_0 increases. This is intuitive because as E_s/N_0 increases a single frame offset will often have a probability greater than $(1-P_{ex})$. In general since an integer number of offsets must be chosen, the actual probability contained in the $(1-P_{ex})$ HPD region will be greater than $(1-P_{ex})$. We observe error floors for the 7 and 15 length sync words due to sequences identical or nearly identical to the sync word being generated in the data symbols. As expected the error floors occur at approximately $1-(1-2^{-L})^K$. The length 31 and 63 *m*-sequences are able to achieve our target performance of 10^{-4}. The values of E_s/N_0 at which the FSER crosses 10^{-4} are recorded in Table I.

Fig. 2 shows the average HPD size for $P_{ex}=10^{-4}$. For each *m*-sequence length there is a value of E_s/N_0 below which the average HPD region size becomes very large. The E_s/N_0 threshold at which the average size of the HPD region crosses 2 is recorded in Table I for each *m*-sequence. The values of E_s/N_0 in Table I provide ranges over which the code-aided method can provide an improvement in performance with a reasonable increase in complexity at the receiver. The code-aided method is able to provide an improvement in performance even above the upper limit values and this comes at almost no cost in terms of complexity (the average HPD size approaches one). The exception here is the length 7 *m*-sequence. Due to the error floor for this sync word, the HPD region is limited to an average size of 8.8 for high E_s/N_0. In Table I 1.5 dB and 2.5 dB intervals are identified for the length 63 and 31 *m*-sequences, respectively. Additionally, the code-aided approach is useful for any E_s/N_0 over 3 dB for the length 15 *m*-sequence. The success of the code-aided method depends on its ability to choose the correct offset from the HPD region which we consider next.

Simulations are presented for a receiver implementing the proposed method. The simulation parameters are compiled in Table II. The frame synchronization and data detection results are shown in Fig. 3. The target P_{ex} is 1/10th the code FER with perfect synchronization. As expected, the simulated P_{ex} is just below the targeted P_{ex}. The FSER of the proposed receiver is very close to the P_{ex} and provides significant gains

TABLE I. COMPLEXITY AND PERFORMANCE BOUNDS

m-seq. length	E_s/N_0 for which ave. HPD size < 2	E_s/N_0 for which uncoded FSER $< 10^{-4}$
7	7 dB	-
15	3 dB	-
31	-1.5 dB	1 dB
63	-5 dB	-3.5 dB

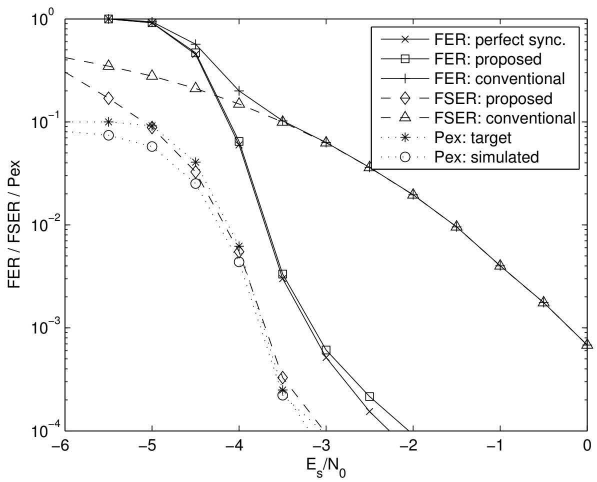

Fig. 3. Frame synchronization and data detection performance of the proposed receiver for $L = 31$. Note that $E_b/N_0 = E_s/N_0 + 4.8$ dB as a result of the 1/3-rate code.

over the FSER of the conventional receiver. The proposed receiver has a slight loss in FER performance below 10^{-3} due to synchronization errors. The proposed receiver has a gain of 2 dB or more over the conventional receiver for FERs below 10^{-2}. From Fig. 1 the FSER of the length 63 m-sequence is at least one order of magnitude below the 'perfect sync' FER of the 1/3-rate code. Using the length 63 m-sequence would also achieve reliable performance but at the cost of doubling the overhead of the sync word.

The average HPD size is shown in Fig. 4. Recall that the target P_{ex} is 1/10th the code FER (up to a limit of 10^{-4}). Since the FER of the code falls quickly with E_s/N_0, the HPD actually increases in average size with E_s/N_0 during the 'waterfall' region of the code. We found that running a single iteration of the decoder is sufficient to perform reliable code-aided frame synchronization. For data detection 10 iterations are performed. Complexity, measured as the average number of decoder iterations that must be performed, is shown in Fig. 4. As an example, for an average HPD size of 5, an average of 5 iterations are required for synchronization and an additional 9 are required for detection which results in a complexity 1.4 times that of the conventional receiver.

III. FRAME SYNCHRONIZATION: SERIAL APPROACH

In this section, we explore serial processing of the frame offsets where a binary hypothesis test is used to determine whether the frame is sync'ed or not sync'ed. For the null hypothesis $\mathcal{H}_0$—the case in which the frame is not synchronized—the symbols are modeled as iid and uniformly distributed over the constellation space. For the alternative

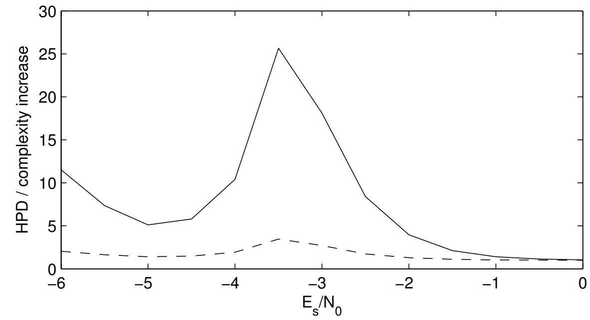

Fig. 4. Average number of frame offsets in the HPD region (solid line) and complexity increase a a multiple of the complexity of a conventional receiver which performs 10 decoding iterations (dashed line).

TABLE II. SIMULATION PARAMETERS

Modulation	BPSK
Code	1/3-rate PCCC (Turbo code) poly. generators $\{23, 35\}$
Interleaver	Bit (turbo code) and symbol interleavers: pseudo-random permutations
Sync word	m-sequence $L = 31$
No. information bits	512
No. coded symbols	$K = 1544$

hypothesis $\mathcal{H}_1$—the case in which the frame is synchronized—the symbols are modeled as being generated from a sync word and a modulated codeword. Thus, in the alternative hypothesis, the code structure is present in the received samples.

In this approach, detection is performed on N samples of the received sequence $\mathbf{y} = [y_0, \ldots, y_{N-1}]$. The likelihood of the null hypothesis is given by

$$f(\mathbf{y}|\mathcal{H}_0) = \sum_{\mathbf{x}} f(\mathbf{y}|\mathbf{x}) p(\mathbf{x}|\mathcal{H}_0) = \prod_{i=0}^{N-1} \sum_{x_i \in \mathcal{X}} f(y_i|x_i) \frac{1}{M}. \quad (8)$$

Due to the use of m-sequences and a symbol interleaver, the assumption that the symbols are iid in the *null hypothesis* is a valid approximation.

The likelihood function for the alternative hypothesis is given by

$$f(\mathbf{y}|\mathcal{H}_1) = \sum_{\mathbf{x}} f(\mathbf{y}|\mathbf{x}) p(\mathbf{x}|\mathcal{H}_1) = \sum_{\mathbf{a}} f(\mathbf{y}|\mathbf{s}, \mathbf{a}) p(\mathbf{a}|\mathcal{H}_1) \quad (9)$$

where the symbols are modeled as a synchronized frame $\mathbf{x} = [\mathbf{s}^{\mathrm{T}} \mathbf{a}^{\mathrm{T}}]^{\mathrm{T}}$. Let $\mathbf{c}$ denote the coded bits and $\mathbf{b}$ denote the information bits. The likelihood function can be further expanded as given by

$$f(\mathbf{y}|\mathcal{H}_1) = \sum_{\mathbf{a}} \sum_{\mathbf{c}} \sum_{\mathbf{b}} f(\mathbf{y}|\mathbf{s}, \mathbf{a}) p(\mathbf{a}|\mathbf{c}, \mathcal{H}_1) p(\mathbf{c}|\mathbf{b}, \mathcal{H}_1). \quad (10)$$

The computation of (10) requires summation over all codewords which is impractical. The factor graph representation of (10) contains cycles and so exact marginalization is not possible. Even so, iterative algorithms have been shown to effectively perform the marginalization in (10). We apply the sum product algorithm operating on the factor graph of the joint distribution $f(\mathbf{y}, \mathbf{s}, \mathbf{a}, \mathbf{c}, \mathbf{b})$ to iteratively approximate the marginalization in (10). For an introduction to factor graphs and the sum-product algorithm please refer to [15].

The optimal decision rule is given by

$$\ln \frac{f(\mathbf{y}|\mathcal{H}_1)}{f(\mathbf{y}|\mathcal{H}_0)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \lambda. \tag{11}$$

In order to reduce the complexity of this method, we may first considered the confidence of a detector constructed for an uncoded case. The optimum decision rule for detection of the sync word for BPSK modulation is given by

$$\ln \frac{f(y_0, \ldots, y_{L-1}|\mathcal{H}_1)}{f(y_0, \ldots, y_{L-1}|\mathcal{H}_0)} \propto$$

$$\frac{2}{N_0} \sum_{i=0}^{L-1} y_i s_i - \sum_{i=0}^{L-1} \ln \cosh\left(\frac{2}{N_0} y_i\right) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \lambda. \tag{12}$$

In the receiver, the sync word detector is first applied to determine whether the coded-aided decision rule is needed. When the sync word detector has a high confidence that the frame is not synchronized (a negative value for the log-likelihood), this frame offset may be "skipped" by the high-complexity code-aided decision rule.

A. Numerical Results

The simulation parameters are given in Table II. Through simulation, it was found that a threshold of -5 for the sync word decision rule and a threshold of 0 (ML detection) for the coded decision rule produced good performance. The probability of false alarm P_{FA} and the probability of missed detection P_{MD} are shown in Fig. 5. The performance is compared to ML detection using the sync word decision rule. The detector achieves error rates 1-2 orders of magnitude below the FER of the code. As a reference for the ability of the sync word detector to reduce complexity, the false alarm rate of the sync word detection rule is 4.3% for $E_s/N_0 = -3.5$ dB. Thus, the high complexity coded-aided synchronization is avoided in 95.7% of cases in which the frame is not synchronized.

IV. Conclusion

In this paper, we considered parallel and serial approaches to code-aided frame synchronization. Complexity in the estimation approach was reduced by considering only the frame offsets contained in the HPD region. It was shown that code-aided method provide a range of SNR values over which the performance was improved with respect to conventional synchronization. Due to the dynamic selection of frame offsets using the HPD region, the complexity of the code-aided method approaches that of the conventional receiver as SNR is increased. The serial approach was motivated out the need to reliably determine whether a frame was synchronized without processing all possible offsets. In this case, the complexity was reduced by implementing a low-complexity decision rule based on the sync word only. The detector achieved error rates 1-2 orders of magnitude below the FER of the code.

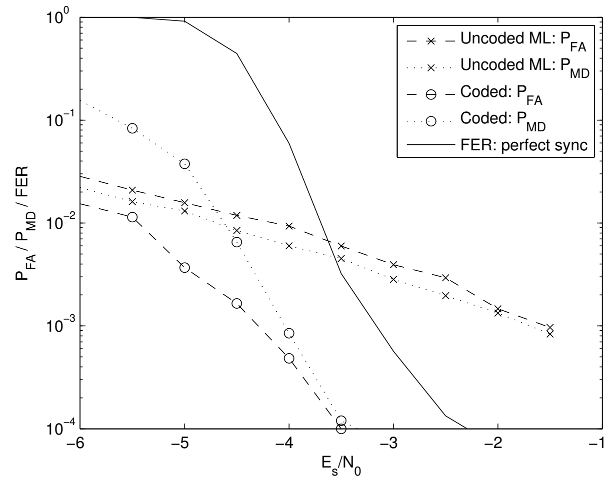

Fig. 5. Performance of the serial frame detection method

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