Acoustics, Speech and Signal Processing (ICASSP), 2013 IEEE International Conference on

Asynchronous Time-of-Arrival-Based Source Localization

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***Abstract*—In this paper, asynchronous wireless source localization using time-of-arrival (TOA) measurements is studied. In TOA localization, the travel time of the signal between the source node and anchor nodes is measured and used to estimate range. In synchronous networks, the anchor nodes know when the source node starts transmission. In asynchronous networks, however, the source transmit time is unknown and TOA measurements have a positive bias due to the synchronization error which could lead to a large localization error. One way to tackle this problem is to use time-difference-of-arrival (TDOA) measurements which do not depend on the source transmission time. However, in this work, applying an alternative approach, we estimate the source transmit time as a nuisance parameter jointly with the source location. The optimal maximum likelihood (ML) estimator is derived. To avoid the ML convergence problem, a novel semidefinite programming (SDP) technique is proposed by converting the noncovex ML problem into a convex one. Computer simulations showing superior performance of the proposed SDP estimator are conducted.**

***Index Terms*—time-of-arrival (TOA), source localization, semidefinite programming (SDP), asynchronous networks.**

I. Introduction

Wireless sensor networks (WSN) have recently been proposed in a wide range of military, commercial, and industrial applications. One of the most important issues in WSN is that the location of each sensor should be determined. However, it would be expensive and impractical to equip all sensors in the network with a Global Positioning System (GPS) receiver. Moreover, GPS makes sensors bulkier and uses more energy. As a result, the concept of source localization has emerged in which sensors are localized by using noisy measurements among themselves [1]–[3]. In source localization, there are some anchor nodes with known locations which localize the source node with an unknown location using noisy measurements. Different measurement techniques typically used in source localization include time-of-arrival (TOA) [4], time-difference-of-arrival (TDOA) [5], received signal strength (RSS) [6], [7], and angle of arrival [8].

In TOA, anchor nodes measure the travel time of the signal from the source node [1]. TOA requires the source node and anchor nodes to be synchronized perfectly which means that the anchor nodes should know when the source node starts its transmission [1]. Although the ease of implementation as well as the high accuracy when using wide-band signals has led TOA to be used in WSN [3], synchronization in WSN is more challenging and makes the network more complicated. There are generally three approaches to deal with the networks where the sensors are not synchronized [1]. First, one can use a two-way TOA technique in which the anchor node transmits a signal and measures the round trip travel time from the anchor node to the source node [9]. However, in two-way TOA, the source node should be able to send and receive data, unlike the classic TOA technique in which the source node is only required to send data. Moreover, the internal delay of the source node should be taken into account which may be different for each sensor [1]. Second, TDOA provides another solution for asynchronous networks [5], [10], [11]. In TDOA, an anchor node is selected as a reference and its TOA measurement is subtracted from the TOA measurements of other anchor nodes which removes the dependency of the measurements on the source transmit time. However, this approach enhances correlation among the measurements which can lead to accuracy degradation [2]. Last but not least, one can estimate the source transmit time jointly with the source location by directly using TOA measurements [12]. In this work, we concentrate on the last approach to deal with asynchronous networks. It should be noted, the last two techniques require synchronization among anchor nodes rather than between the source node and the anchor nodes. Moreover, no matter which of the above techniques is used, the accuracy of the localization degrades in asynchronous networks in comparison with synchronous ones. This is the cost that must be paid for the lack of synchronization between the source and the anchor nodes in the network.

The Cramér-Rao lower bound (CRLB) of TOA localization is derived in [1]. The maximum likelihood (ML) estimator provides the optimal accuracy asymptotically, meaning that it can attain the CRLB for sufficiently high signal-to-noise ratio (SNR) [13]. However, the cost function of the ML estimator is highly nonlinear and nonconvex [4], [14] and does not have a closed-form solution. However, the solution of the ML estimator can be approximately found by using iterative algorithms [15]. The problem is that iterative algorithms require a good starting point to guarantee that the estimator converges to the global minimum. Even with a good initial point, the algorithm may converge to either a local minimum or a saddle point introducing a large estimation error. Linearization and convex relaxation have been applied to tackle the convergence problem of the ML estimator. The performance of linear estimators is poor, especially when either the number of available anchor nodes is limited or the source node is located outside the convex hull of the anchor nodes [10], [16]. Convex relaxation provides reasonably higher accuracy, although the complexity increases in comparison with linear estimators [5], [11], [17]–[19].

In this work, asynchronous TOA-based source localization using semidefinite programming (SDP) technique is studied. SDP is a form of convex optimization which unlike the nonconvex ML estimator does not have convergence problems. An SDP estimator using TDOA measurements is derived in [5]. Here, considering an alternative solution for asynchronous TOA networks, we proposed an SDP estimator which tries to estimate the source transmit time jointly with the source location. Although the problem was previously considered in [5], [17], exploiting a different relaxation technique the proposed SDP estimator has considerably higher accuracy and lower complexity than the SDP estimators in [17] and [5]. Unlike previous studies, moreover, the performance of the proposed SDP and previously studied estimators is evaluated under non-line-of-sight (NLOS) propagation which significantly affects the accuracy of localization in indoor environments [20]–[22].

The following notation is used through the paper. Lowercase and uppercase letters denote scalar values. Bold uppercase and bold lowercase letters denote matrices and vectors, respectively. $\|\cdot\|_2$ denotes the ℓ_2 norm. $\mathbf{I}_M$ and $\mathbf{0}_M$ denote the $M \times M$ identity and the $M \times M$ zero matrices, respectively. For arbitrary symmetric matrices $\mathbf{A}$ and $\mathbf{B}$, $\mathbf{A} \succeq \mathbf{B}$ means that $\mathbf{A} - \mathbf{B}$ is positive semidefinite.

II. System Model

In this section, the measurement model of asynchronous TOA localization will be described. We consider a network with M anchor nodes whose locations are known and one source node with an unknown location. Denote by $\mathbf{y}_i \in \mathbb{R}^2$, $i = 1, 2, \ldots, M$ the location of the ith anchor node and by $\mathbf{x} \in \mathbb{R}^2$ the location the source node to be estimated. The measured TOA at the ith anchor node is modeled as [1], [2]

$$t_i = \frac{d_i}{c} + \tau + v_i, \quad i = 1, 2, \ldots, M, \tag{1}$$

where c is the propagation velocity of the signal depending on the environment where the network is placed (e.g., the propagation velocity in free space is $c \approx 3 \times 10^8$ m/s). v_i is the measurement noise modeled as a zero-mean Gaussian random variable and $d_i = \|\mathbf{x} - \mathbf{y}_i\|_2$ is the true distance between the source node and the ith anchor node. τ is the source transmit time appeared in the model due to lack of synchronization between the source node and anchor nodes. In synchronous networks, the anchor nodes know when exactly the source node transmit the signal and τ is available to the estimator. Therefore, bias-free TOA measurements can be simply obtained by subtracting τ from all measurements. However, in this work, we assume that sensors are asynchronous where the transmit time is unknown and should be estimated. For simplicity, all obtained TOA measurements are multiplied by the propagation velocity to obtain distance information as

$$r_i = ct_i = d_i + d_0 + n_i, \tag{2}$$

where r_i would be the measured distance. $d_0 = c\tau$ is an unknown distance added to the measured distance due to unknown transmit time and $n_i = cv_i$ is the measurement error. Since the accuracy of TOA measurements is related to the received SNR [1], [23] which itself is a function to the distance and path-loss exponent [6], [24], the variance of n_i is typically modeled as [22]

$$\sigma_i^2 = \alpha d_i^{\beta}, \tag{3}$$

where α is a constant which defines the relationship between the noise variance and the true distance and depends on the environment where the network is placed. β is the path-loss exponent which typically varies between 2 and 4 [22]. Again, we assume that the anchor nodes are synchronized in (1).

III. CRLB and Maximum Likelihood Estimation

The CRLB determines a lower bound on the performance (variance) of any unbiased estimator [13, Ch. 3]. The CRLB of the unknown parameters, $\boldsymbol{\theta} = [\mathbf{x}^{\mathrm{T}} d_0]^{\mathrm{T}}$, is obtained from the diagonal elements of the inverse of the Fisher information matrix [13, Ch. 3]. Denote by $\mathbf{r} = [r_1, r_2, \ldots, r_M]^{\mathrm{T}}$ the vector of all measurements and by $\boldsymbol{\mu}(\boldsymbol{\theta})$ the mean of the vector $\mathbf{r}$. The Fisher information matrix of the model in (2) is calculated as [13, Ch. 3]

$$\mathbf{I}(\boldsymbol{\theta}) = \mathbf{F}(\boldsymbol{\theta})^{\mathrm{T}} \mathbf{W} \mathbf{F}(\boldsymbol{\theta}), \tag{4}$$

where $\mathbf{W} = \mathrm{diag}\{\sigma_1^{-2}, \sigma_2^{-2}, \ldots, \sigma_M^{-2}\}$ and

$$\mathbf{F}(\boldsymbol{\theta}) = \frac{\partial \boldsymbol{\mu}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} = \begin{bmatrix} \frac{(\mathbf{x} - \mathbf{y}_1)^{\mathrm{T}}}{d_1} & 1 \\ \vdots & \\ \frac{(\mathbf{x} - \mathbf{y}_M)^{\mathrm{T}}}{d_M} & 1 \end{bmatrix}.$$

The CRLB of the unknown parameter $\boldsymbol{\theta}$ is computed as

$$\mathrm{Var}([\boldsymbol{\theta}]_r) \geq \left[\mathbf{I}^{-1}(\boldsymbol{\theta})\right]_{r,r}. \quad r = 1, 2, 3. \tag{5}$$

When the number of measurements tends to infinity, the ML estimator can achieve the CRLB [13]. In other words, the ML estimator is asymptotically optimal. The ML estimator of the measurement model in (2) is obtained by the following optimization problem [13]

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta} \in \mathbb{R}^3}{\arg\min} \sum_{i=1}^{M} \sigma_i^{-2} \left(r_i - d_i - d_0\right)^2. \tag{6}$$

As mentioned in introduction, the cost function in (6) is severely nonlinear and nonconvex, and does not have a closed-form solution. The solution of the ML estimator can be approximately found by iterative numerical techniques such as the Gauss-Newton method [13], [15]. Such iterative algorithms require a good initialization so that the algorithm converges to the global minimum. However, even with a good starting point, the iterative solver of the ML estimator may return a local minimum or saddle point which can cause a large estimation error.

IV. Semidefinite Programming

In this section, the derivation of the proposed SDP approach is described. First, the nonlinear cost function of the ML estimator is converted into a convex cost function and then is formulated as a SDP optimization problem. Unlike the ML estimator, the proposed SDP technique neither requires initialization nor has convergence problems [25], [26].

The cost function of the ML estimator in (6) can be alternatively written as

$$\begin{aligned}&(\mathbf{r}-\mathbf{d}-\mathbf{d}_0)^{\mathrm{T}}\mathbf{W}(\mathbf{r}-\mathbf{d}-\mathbf{d}_0)=\\&\mathrm{Trace}\left\{\mathbf{W}\Big(\mathbf{r}-(\mathbf{d}+\mathbf{d}_0)\Big)\Big(\mathbf{r}-(\mathbf{d}+\mathbf{d}_0)\Big)^{\mathrm{T}}\right\},\end{aligned}\tag{7}$$

where $\mathbf{d}=[d_1,d_2,\ldots,d_M]^{\mathrm{T}}$ and $\mathbf{d}_0=[d_0,d_0,\ldots,d_0]^{\mathrm{T}}$. Defining a new vector as $\mathbf{h}=[d_1,d_2,\ldots,d_M,d_0]^{\mathrm{T}}$, we can write

$$\mathbf{d}+\mathbf{d}_0=\mathbf{U}\mathbf{h},\tag{8}$$

where $\mathbf{U}=[\mathbf{I}_M,\mathbf{1}_M]$. Plugging (8) in (7) yields

$$\begin{aligned}&\mathrm{Trace}\left\{\mathbf{W}(\mathbf{r}-\mathbf{U}\mathbf{h})(\mathbf{r}-\mathbf{U}\mathbf{h})^{\mathrm{T}}\right\}=\\&\mathrm{Trace}\left\{\mathbf{W}\left(\mathbf{r}\mathbf{r}^{\mathrm{T}}-2\mathbf{U}\mathbf{h}\mathbf{r}^{\mathrm{T}}+\mathbf{U}\mathbf{H}\mathbf{U}^{\mathrm{T}}\right)\right\},\end{aligned}\tag{9}$$

where $\mathbf{H}=\mathbf{h}\mathbf{h}^{\mathrm{T}}$. The diagonal elements of the matrix $\mathbf{H}$ are

$$[\mathbf{H}]_{ii}=d_i^2=\begin{bmatrix}\mathbf{y}_i\\-1\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\mathbf{I}_2&\mathbf{x}\\\mathbf{x}^{\mathrm{T}}&z\end{bmatrix}\begin{bmatrix}\mathbf{y}_i\\-1\end{bmatrix},i=1,2,\ldots,M,\tag{10}$$

where $z=\mathbf{x}^{\mathrm{T}}\mathbf{x}$. To convert the nonconvex cost function in (9) into a convex function, we have to relax non-affine operations. By relaxing the matrix $\mathbf{H}$ and the variable z, they can be written as a linear matrix inequality (LMI) using Schur complement [5], [25]

$$\begin{aligned}z=\mathbf{x}^{\mathrm{T}}\mathbf{x}&\;\Rightarrow\begin{bmatrix}\mathbf{I}_2&\mathbf{x}\\\mathbf{x}^{\mathrm{T}}&z\end{bmatrix}\succeq\mathbf{0}_3,\\\mathbf{H}=\mathbf{h}\mathbf{h}^{\mathrm{T}}&\;\Rightarrow\begin{bmatrix}\mathbf{H}&\mathbf{h}\\\mathbf{h}^{\mathrm{T}}&1\end{bmatrix}\succeq\mathbf{0}_{M+2}.\end{aligned}\tag{11}$$

Therefore, the nonlinear and nonconvex ML problem of (6) can be relaxed into an SDP optimization problem as [25]

$$\begin{aligned}\underset{\mathbf{x},z,\mathbf{h},\mathbf{H}}{\text{minimize}}\quad&\mathrm{Trace}\left\{\mathbf{W}(\mathbf{U}\mathbf{H}\mathbf{U}^{\mathrm{T}}-2\mathbf{U}\mathbf{h}\mathbf{r}^{\mathrm{T}}+\mathbf{r}\mathbf{r}^{\mathrm{T}})\right\}\\\text{subject to}\quad&[\mathbf{H}]_{ii}=\begin{bmatrix}\mathbf{y}_i\\-1\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\mathbf{I}_2&\mathbf{x}\\\mathbf{x}^{\mathrm{T}}&z\end{bmatrix}\begin{bmatrix}\mathbf{y}_i\\-1\end{bmatrix},\\&\begin{bmatrix}\mathbf{H}&\mathbf{h}\\\mathbf{h}^{\mathrm{T}}&1\end{bmatrix}\succeq\mathbf{0}_{M+2},\;\begin{bmatrix}\mathbf{I}_2&\mathbf{x}\\\mathbf{x}^{\mathrm{T}}&z\end{bmatrix}\succeq\mathbf{0}_3.\end{aligned}\tag{12}$$

The solution of (12) can be effectively found with the numerical algorithms such as interior point methods [25], [26]. Standard SDP solvers such as SeDuMi [27] is employed to solve SDP optimization problems in MATLAB.

V. Simulation Results

In this section, computer simulations are conducted to evaluate the performance of the proposed SDP estimator. TOA measurements were generated based on the measurement model in (1). The propagation speed was set to 3×10^8 m/s. The values of β and α were set to 2 and 0.05, respectively. The source transmit time, τ, was drawn from a uniform distribution $\mathcal{U}[3.3,10.3]$ ns which leads to d_0 varying from 1 to 4 m. The ML estimator and four previously considered estimators were selected for comparison. An SDP estimator using TDOA measurements is derived in [5]. Two other SDP estimators are derived in [17] which directly use asynchronous TOA measurements. A well-known linear least squares (LLS) estimator using TDOA measurements is derived in [10]. A summary of the compared estimators is given in Table I. The ML estimators in (6) and [28] were solved by the MATLAB routine `fminunc` and were initialized with the true values. Thus, these represent performance not truly attainable by ML, but close to optimal. The proposed SDP and three other SDP estimators were implemented by the `CVX` toolbox [29] using SeDuMi as a solver [27]. The LLS estimator in [10] has a closed-form solution.

A network with eight anchor nodes and one source node was considered. The locations of the anchor nodes are fixed and 441 different locations for the source node are generated uniformly in a square region of 10m × 10m. Fig. 1 shows the configuration of the simulated network. For each source location, 100 Monte Carlo realizations were done. The cumulative distribution function (CDF) of localization

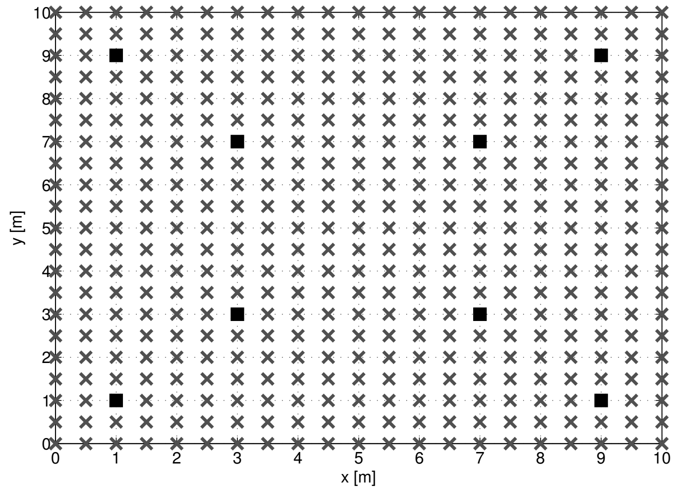

Fig. 1. The configuration of the simulated network. The solid squares and the crosses indicate the anchor nodes the locations of the source node, respectively.

TABLE I
The average running time of the considered estimators.

Estimator	Description	Time [ms]
ML-TDOA	The ML estimator using TDOA in [28]	19.08
ML-TOA-A	The ML estimator in (6)	29.52
SDP-NEW	The proposed SDP estimator in (12)	60.01
SDP-TDOA	The SDP estimator in using TDOA [5]	104.48
SDP-2LS	The SDP estimator in [17]	111.58
SDP-MMA	The SDP estimator in [17]	37.76
LLS	The linear estimator using TDOA in [10]	0.68

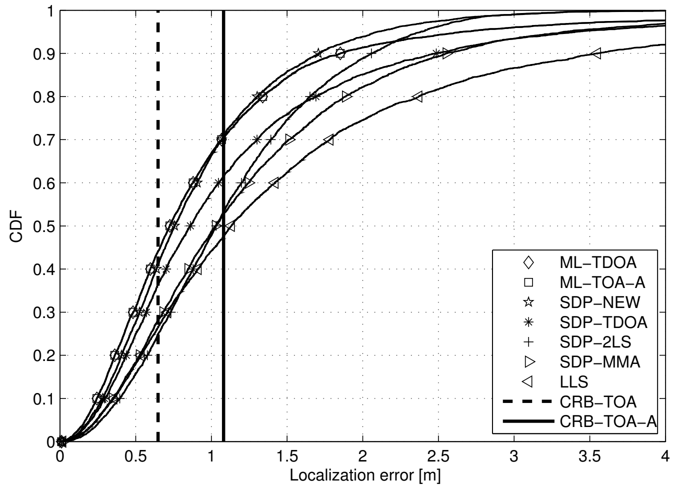

Fig. 2. The CDF of localization error of the compared estimators in a LOS environment. The proposed SDP estimator outperforms other estimators and is very close to the optimal ML estimator.

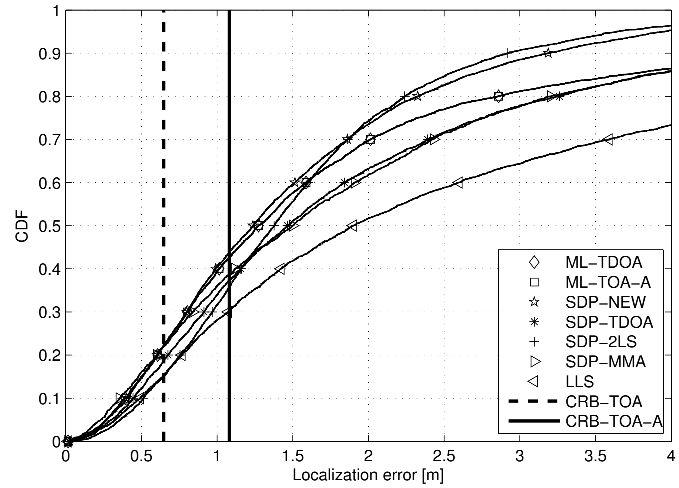

Fig. 3. The CDF of localization error for the compared estimators in a NLOS environment. The performance of all estimator degrades in NLOS environments. However, the proposed SDP estimator shows strong robustness against NLOS propagation.

error of the compared estimators is depicted in Fig. 2. The depicted CRLB is obtained by averaging over the CRLB of all source locations. The average running time of the compared estimators for this network is also given in Table I.

There are two important facts in Fig. 2 that should be noted. First, the accuracy of localization degrades in asynchronous networks in comparison with synchronous ones. The CRLB of asynchronous TOA (labeled as CRLB-TOA-A) is about 60% worse than the CRLB of synchronous TOA [1] (labeled as CRLB-TOA) for this network configuration. Second, the ML estimator estimating the source transmit time jointly with the source location (ML-TOA-A) yields the same localization performance as the ML estimator using TDOA measurements (ML-TDOA). Therefore, no matter which asynchronous technique is used, the optimal ML estimator gives the same estimate for the source location, since they both use the same information. However, we will later show that the situation is different for sub-optimal estimators (e.g., LLS and SDP). In fact, the performance of the sub-optimal estimators depends on which asynchronous technique is used. Although it has not been shown in Fig. 2, we would like to add that based on our simulations the performance of ML-TDOA does not depend on which anchor node is selected as the reference. Table I shows that ML-TDOA has lower running time than ML-TOA-A. The first reason is that the ML-TDOA uses $M-1$ measurements, while ML-TOA uses M measurements. Another reason is that ML-TDOA requires a search over a two-dimensional variable, while ML-TOA-A requires a search over a three-dimensional variable. Therefore, if the optimal estimator is chosen, it seems reasonable to use TDOA.

The proposed SDP estimator (SDP-NEW) provides a remarkable performance and outperforms the other estimators. The performance of SDP-TDOA is also satisfactory but not as good as the proposed SDP estimator. As can be seen, unlike the optimal estimators, SDP-NEW estimating the source transmit time and source location provides higher accuracy than SDP-TDOA using TDOA measurements. Moreover, it can be shown that unlike ML-TDOA, the performance of SDP-TDOA depends on which anchor node is selected as a reference, since the correlation among measurements is neglected in SDP-TDOA [5]. It was shown in [30] that the anchor node whose TOA measurement is the median of all TOA measurements should be selected as the reference. The same criterion was used here for SDP-TDOA. Another drawback of SDP-TDOA is its complexity. Table I also shows that the running time of SDP-TDOA is almost double that of SDP-NEW, highlighting another advantage of the proposed SDP estimator over SDP-TDOA. Fig. 2 also shows that SDP-NEW outperforms the ML estimator for localization errors larger than 1.25 m. The larger errors in the ML estimator curve belong to the locations of the source node that are outside the convex hull (especially behind the anchor nodes, e.g., $[10, 10]^{\mathrm{T}}$). At these locations, the cost function of the ML estimator is very sensitive to the initialization and even though the estimator was initialized with true values, it returned a local minimum causing a large localization error. This is the major problem of the ML estimator that leads us to use alternative estimators which either are closed-from or do not require initialization. Both ML estimators have lower running time than SDP-NEW. However, it should be noted that the solvers of the ML estimators are initialized with true values which decreases their running times considerably [7].

Both SDP-2LS and SDP-MMA estimators [17] directly use asynchronous TOA measurements. However, their performance is not as good as SDP-NEW. SDP-2LS requires a tuning parameter on which the performance of the estimator highly depends [17]. As a result, for each network configuration, it is required to find the optimum calibration parameter in order to achieve good accuracy. SDP-MMA is based on a min-max formulation which generally has lower accuracy than minimum mean square estimation [26], although it has lower complexity. On the other hand, in the formulation of SDP-MMA, it is required to square the measurements which causes noise enhancement in the estimator [17]. LLS has the worst accuracy among the considered estimators mainly because of simplistic approximations [10], although it has the fastest running time among the compared estimators. It should be noted that our observations show that among compared estimators, SDP-NEW and SDP-2LS are mildly sensitive to the value of τ. In fact, as τ increases the performance of SDP-NEW degrades slightly. To provide a numerical example, when the range of τ was increased by a factor of 4 (i.e., d_0 varies from 1 to 16 m), the accuracy of SDP-NEW degraded 0.1 m at 60% CDF which is not significant.

Source localization is more useful in indoor environments where GPS receivers do not work properly. However, the majority of the available connections in indoor networks are NLOS because of several obstructions between the source and anchor nodes [31]. In Fig. 3, the sensitivity of the compared estimators to NLOS propagation is evaluated. The same configuration as Fig. 2 was considered except two out of eight connections were NLOS in this case [20]–[22]. A NLOS link was generated by adding a large positive bias to its range estimate. NLOS biases were drawn from an exponential distribution where its mean was randomly selected from 1 to 4 m. As shown in Fig. 3, the performance of all estimators degrades in NLOS environments. However, the performances of the estimators that estimate the source transmit time are less sensitive to NLOS propagation than the estimators that use TDOA measurements, since some parts of NLOS biases are compensated in the estimate of the source transmit time.

VI. Conclusion

Asynchronous TOA-based wireless source localization was examined. In an asynchronous network, the transmit time of the source node is not known to the anchor nodes, which causes an unknown bias to appear in the TOA measurement model. It was shown that localization accuracy degrades in asynchronous networks in comparison with synchronous ones. To avoid the maximum likelihood (ML) estimator convergence problem, a novel semidefinite programming (SDP) estimator estimating the source transmit time jointly with the source location was derived. Simulation results showed that the proposed SDP estimator requiring no initialization provides remarkable performance which is very close to the optimal accuracy with a satisfactory complexity.

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