Received Signal Strength-Based Sensor Localization in Spatially Correlated Shadowing

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***Abstract*—Wireless sensor localization using received signal strength (RSS) measurements is investigated in this paper. Most studies for RSS localization assume that the shadowing components are uncorrelated. However in this paper, we assume that the shadowing is spatially correlated. Under this condition, it can be shown that the localization accuracy can be improved if the correlation among links is taken into consideration. Avoiding the maximum likelihood (ML) convergence problem, we derive a novel semidefinite programming (SDP) approach by converting the corresponding noncovex ML estimator into a convex one. The performance of the proposed SDP estimator is compared with the ML estimator and previously considered estimators. Computer simulations show that the proposed SDP estimator outperforms the previously considered estimators in both uncorrelated and correlated shadowing environments.**

***Index Terms*—received signal strength (RSS), sensor localization, correlated shadowing, semidefinite programming (SDP).**

I. Introduction

Recently, position location has been widely studied in the literature because of its important applications in both wireless sensor networks (WSN) and cellular systems. In a WSN, the collected information from the sensors is often useless if their positions are not available. However, equipping all sensors with a Global Positioning System (GPS) receiver is very expensive and typically impractical. Moreover, GPS does not work properly in indoor environments or forest environments where the satellite signal attenuates severely. Sensor localization generally requires two or more anchor nodes with known positions to determine the location of source nodes with unknown positions via noisy measurements such as time-of-arrival (TOA) [1], [2], received signal strength (RSS) [3]–[5], and time-difference-of-arrival (TDOA) [6]. In this paper, RSS localization is considered, since it is practically simple and inexpensive to implement [1].

The Cramér-Rao lower bound (CRLB) of RSS localization was derived in [3], [7]. The maximum likelihood (ML) estimator is always one of the most interesting estimators because it can attain the CRLB accuracy for sufficiently high signal-to-noise-ratio. The ML estimator of RSS localization was derived in [3]. It can be shown that the cost function of the ML estimator is highly nonlinear and nonconvex [8], [9]. The ML estimator does not have a closed-form solution, but it can be approximately solved by iterative algorithms [10], [11]. However, iterative algorithms require a good starting point to make sure that they converge to a global minimum. If an appropriate initialization is not selected for an iterative algorithm, it can get stuck in either a local minimum or a saddle point which can introduce a large estimation error. Sub-optimal estimators such as linear estimators and convex estimators have emerged to deal with the convergence problem of the ML estimator. A linear least squares (LLS) estimator for RSS localization is derived in [4]. Although linear estimators have a closed-form solution, their performance is not as accurate as the ML estimator, especially when the number of available anchor nodes is limited [9]. Convex relaxation is another solution for the ML problem in which the nonconvex cost function of the ML estimator is converted into a convex optimization problem. Semidefinite programming (SDP) [8], [12]–[14] and second order cone programming (SOCP) [15] are the convex estimators that are typically considered for sensor localization. In [13], assuming pair-wise distance measurements are available, an SDP estimator for the estimation of the source location was derived. However, in RSS localization, the received powers are measured and the pair-wise distances are not directly available. An SDP estimator which is directly applied to the RSS measurements was derived in [8], [16].

Most previous studies assume that the RSS measurements are uncorrelated. However, in most indoor environments, the shadowing components are generally correlated due to physical obstructions. More specifically, neighboring sensor nodes often experience the same shadowing which makes RSS links correlated [7]. It will be shown in this paper that when there is correlation among RSS measurements, the accuracy of positioning can be improved, which is consistent with the results in [7], [17]. However, if a localization algorithm neglects the correlation, the improvement is not significant.

In this paper, RSS sensor localization in spatially correlated shadowing is examined. First, we derive the corresponding ML estimator for the measurement model. Then, by using a Taylor series expansion and convex relaxation, the ML cost function is converted into an SDP optimization problem. The performance of the proposed SDP estimator is evaluated through computer simulations in both uncorrelated and correlated shadowing environments.

II. System Model

The measurement model of RSS localization with correlated shadowing will be described in this section. A network with M anchor nodes with known positions and one source node with an unknown position is considered. Let $\mathbf{y}_i \in \mathbb{R}^2$, $i = 1, 2, \dots, M$ be the known coordinates of the ith anchor node and $\mathbf{x} \in \mathbb{R}^2$ be the unknown coordinates of the source node. The received power (in dBm) at the ith anchor node, P_i, under log-normal shadowing is modeled as [3]

$$P_i = P_0 - 10\beta \log_{10} \frac{d_i}{d_0} + n_i, \qquad i = 1, 2, \ldots, M, \tag{1}$$

where P_0 (in dBm) is the reference power at distance d_0 from the source node. β is the path loss exponent which varies typically between 2 and 4 depending on the propagation environment [1]. $d_i = \|\mathbf{x} - \mathbf{y}_i\|_2$ is the true distance between the source node and the ith anchor node, where $\|\cdot\|_2$ denotes the ℓ_2 norm. n_i are the log-normal shadowing terms. In most studies, the shadowing is simply modeled as independent and identically distributed (i.i.d.) zero-mean Gaussian random variables with covariance matrix $\mathbf{Q} = \sigma_{dB}^2 \mathbf{I}_M$ [3], [4], [8], [16]. However, in this paper, we assume that the shadowing is not independent and model the covariance matrix as [7], [17]

$$[\mathbf{Q}]_{ij} = \begin{cases} \sigma_{dB}^2, & \text{if } i = j, \\ \rho_{ij}\sigma_{dB}^2, & \text{if } i \neq j, \end{cases} \tag{2}$$

where σ_{dB} is the standard deviation of the shadowing which is constant with distance and only depends on the propagation environment [1]. Expressed in dB, σ_{dB} is generally between 4 and 12 dB [1]. ρ_{ij} is the correlation coefficient between the ith and the jth links. Unlike most studies which assume $\rho_{ij} = 0$, in practical cases, the shadow fading is spatial correlated ($\rho_{ij} \neq 0$), because of the network configuration and the obstacles between the source and anchor nodes [7], [17]. The correlation between a pair of sensor nodes depends on their relative angles and distances [18]. Empirical studies show that the value of the correlation coefficient typically varies from 0.2 to 0.8 [19]. Without loss of generality, for the rest of the paper, we assume $d_0 = 1$ m. We also assume that the values of P_0, β, and $\mathbf{Q}$ are available to the estimator. However, we do examine the impact of the imperfect $\mathbf{Q}$ on the performance.

III. Maximum Likelihood

The CRLB defines a lower bound on the performance of any unbiased estimator. The ML estimator can achieve the CRLB asymptotically (when the number of measurements tends to infinity) [20]. The ML estimator of the measurement model in (1) is obtained by the following optimization problem [3], [20]

$$\hat{\mathbf{x}}_{\text{ML}} = \underset{\mathbf{x}\in\mathbb{R}^2}{\arg\min} \big(\mathbf{p} - \mathbf{g}(\mathbf{x})\big)^T \mathbf{Q}^{-1} \big(\mathbf{p} - \mathbf{g}(\mathbf{x})\big), \tag{3}$$

where $\mathbf{p} = [P_1, P_2, \ldots, P_M]^T$ is the measurement vector and

$$[\mathbf{g}(\mathbf{x})]_i = P_0 - 10\beta \log_{10} d_i. \tag{4}$$

The objective function in (3) is severely nonlinear and nonconvex, and does not have a closed-form solution. The solution of the ML estimator can be approximately found by iterative numerical techniques such as the Gauss-Newton method [10], [20]. The iterative algorithms require a good initialization to guarantee that the algorithm converges to the global minimum. However, even with a good starting point, the iterative solver of the ML estimator may return a local minimum or saddle point which causes a large estimation error.

IV. Semidefinite Programming

In this section, the proposed SDP estimator will be derived. By using Taylor series expansion, first the original ML estimator is alternatively formulated as a nonlinear least squares (NLS) problem which has a smoother cost function [5]. It is then converted into an SDP optimization problem. Rearranging (1) and dividing both sides by 10β gives

$$\log_{10} d_i + \frac{P_i - P_0}{10\beta} = \frac{n_i}{10\beta}. \tag{5}$$

Taking the power of 10 on both sides yields

$$\lambda_i d_i = 10^{n_i/10\beta}, \tag{6}$$

where $\lambda_i = 10^{(P_i - P_0)/10\beta}$. By using the first-order Taylor series expansion, the right-hand side of (6) can be approximately written for sufficiently small shadowing ($n_i \ll 10\beta/\ln 10$) as [5], [9]

$$\lambda_i d_i = 1 + \frac{\ln 10}{10\beta} n_i. \tag{7}$$

Rearranging (7) yields

$$\lambda_i d_i - 1 = \epsilon_i, \tag{8}$$

where $\epsilon_i = (\ln 10/10\beta) n_i$. Writing (8) in vector form yields

$$\mathbf{L}\mathbf{d} - \mathbf{1}_M = \boldsymbol{\epsilon}, \tag{9}$$

where $\mathbf{L} = \text{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_M\}$, $\mathbf{d} = [d_1, d_2, \ldots, d_M]^T$, and $\boldsymbol{\epsilon} = [\epsilon_1, \epsilon_2, \ldots, \epsilon_M]^T$. The source node location can be estimated by using the weighted NLS of the model in (9) as [20, Ch. 8]

$$\hat{\mathbf{x}} = \underset{\mathbf{x}\in\mathbb{R}^2}{\arg\min}\ \boldsymbol{\epsilon}^T \mathbf{W} \boldsymbol{\epsilon}, \tag{10}$$

where $\mathbf{W}$ is a weighting matrix which is equal to the inverse of the covariance matrix of

$$\mathbf{W} = \big(\text{E}[\boldsymbol{\epsilon}\boldsymbol{\epsilon}^T]\big)^{-1} = (10\beta)^2/(\ln 10)^2 \mathbf{Q}^{-1}. \tag{11}$$

The minimization problem in (10) is still nonlinear and noncovex. The cost function of (10) can be expressed as

$$\begin{aligned} \boldsymbol{\epsilon}^T \mathbf{W} \boldsymbol{\epsilon} &= \text{Trace}\left\{\mathbf{W}\boldsymbol{\epsilon}\boldsymbol{\epsilon}^T\right\}, \\ &= \text{Trace}\left\{\mathbf{W}(\mathbf{L}\mathbf{d} - \mathbf{1}_M)(\mathbf{L}\mathbf{d} - \mathbf{1}_M)^T\right\}, \\ &= \text{Trace}\left\{\mathbf{W}(\mathbf{L}\mathbf{D}\mathbf{L}^T - 2\mathbf{L}\mathbf{d}\mathbf{1}_M^T + \mathbf{1}_M\mathbf{1}_M^T)\right\}, \end{aligned} \tag{12}$$

where $\mathbf{D} = \mathbf{d}\mathbf{d}^T$. The diagonal elements of the matrix $\mathbf{D}$ are

$$[\mathbf{D}]_{ii} = d_i^2 = \begin{bmatrix} \mathbf{y}_i \\ -1 \end{bmatrix}^T \begin{bmatrix} \mathbf{I}_2 & \mathbf{x} \\ \mathbf{x}^T & z \end{bmatrix} \begin{bmatrix} \mathbf{y}_i \\ -1 \end{bmatrix}, \tag{13}$$

where $z = \mathbf{x}^T\mathbf{x}$. To convert the nonconvex objective function in (10) into a convex function, we must relax the elements in (12)-(13) that are not affine (i.e., z and $\mathbf{D}$) [21]. Relaxing non-affine operations, we can write them as a linear matrix inequality (LMI) [22], [23]

$$z = \mathbf{x}^T\mathbf{x} \quad \Rightarrow \quad \begin{bmatrix} \mathbf{I}_2 & \mathbf{x} \\ \mathbf{x}^T & z \end{bmatrix} \succeq \mathbf{0}_3,$$

$$\mathbf{D} = \mathbf{d}\mathbf{d}^T \quad \Rightarrow \quad \begin{bmatrix} \mathbf{D} & \mathbf{d} \\ \mathbf{d}^T & 1 \end{bmatrix} \succeq \mathbf{0}_{M+1}.$$

Finally, the minimization problem of (10) is relaxed into an SDP optimization problem as [22]

$$\begin{aligned} &\underset{\mathbf{x},z,\mathbf{d},\mathbf{D}}{\text{minimize}} \quad \text{Trace}\left\{\mathbf{W}(\mathbf{L}\mathbf{D}\mathbf{L}^T - 2\mathbf{L}\mathbf{d}\mathbf{1}_M^T)\right\} \\ &\text{subject to} \quad [\mathbf{D}]_{ii} = \begin{bmatrix} \mathbf{y}_i \\ -1 \end{bmatrix}^T \begin{bmatrix} \mathbf{I}_2 & \mathbf{x} \\ \mathbf{x}^T & z \end{bmatrix} \begin{bmatrix} \mathbf{y}_i \\ -1 \end{bmatrix}, \\ &\qquad \begin{bmatrix} \mathbf{D} & \mathbf{d} \\ \mathbf{d}^T & 1 \end{bmatrix} \succeq \mathbf{0}_{M+1}, \quad \begin{bmatrix} \mathbf{I}_2 & \mathbf{x} \\ \mathbf{x}^T & z \end{bmatrix} \succeq \mathbf{0}_3. \end{aligned} \tag{14}$$

The solution of (14) can be effectively found with the numerical algorithms such as interior point methods [21], [22]. Unlike the original ML estimator, the proposed SDP estimator does not have convergence problems [21], [22]. Standard SDP solvers such as SeDuMi [24] can be employed to solve SDP optimization problems in MATLAB.

V. Simulation Results

Computer simulations were conducted to evaluate the performance of the proposed estimator. RSS measurements were generated based on the measurement model in (1). The values of the path-loss exponent β and the reference power P_0 were set to 4 [1] and -40 dBm [9], respectively. We assume that all measurements have the same correlation coefficient, hence $\rho_{ij} = \rho$ [17]. The values of σ_{dB}^2 and ρ are mentioned in each figure. The covariance matrix of shadowing, $\mathbf{Q}$, is a symmetric and positive- definite matrix. Then, using Cholesky decomposition, $\mathbf{Q}$ can be decomposed as [17]

$$\mathbf{Q} = \mathbf{B}\mathbf{B}^T, \tag{15}$$

where $\mathbf{B}$ is a lower triangular matrix. Let $\mathbf{n} = [n_1, n_2, \ldots, n_M]^T$ be the vector of shadowing components in (1) which are zero-mean Gaussian random variables with a covariance matrix $\mathbf{Q}$. Then, $\mathbf{n}$ can be generated as [17]

$$\mathbf{n} = \mathbf{B}\mathbf{w}, \tag{16}$$

where $\mathbf{w}$ is a vector of zero-mean iid Gaussian random variables with unit variance. It should be noted that the proposed SDP estimator works for every covariance matrix $\mathbf{Q}$, even if we have different correlation coefficients for the measurements. The ML estimator was solved by MATLAB routine `fminunc` and was initialized with the true value of the source location. The proposed SDP estimator was implemented by `cvx` [25] using SeDuMi as a solver [24].

Besides the ML estimator, three previously proposed estimators were selected for comparison. An SDP estimator was derived in [8] which directly used the RSS measurements. Another SDP estimator was derived in [16] which is based on an unscented transformation and SDP relaxation. Further, we also included a linear least squares (LLS) estimator with a closed-form solution given in [4] in the comparison. A summary of the compared estimators can be found in Table I.

TABLE I
The average running time of the compared estimators. CPU: Intel Core 2 Duo E7500 2.93 GHz.

Estimator	Description	Time [ms]
ML-UNC	The ML estimator in (3) with $\mathbf{Q} = \mathbf{I}_M$	14.83
ML	The ML estimator in (3)	16.24
SDP-NEW	The proposed SDP estimator in (14)	48.06
SDP-RSS	The SDP estimator in [8]	74.68
SDP-UT	The SDP estimator in [16]	52.27
LLS	The linear estimator in [4]	00.28

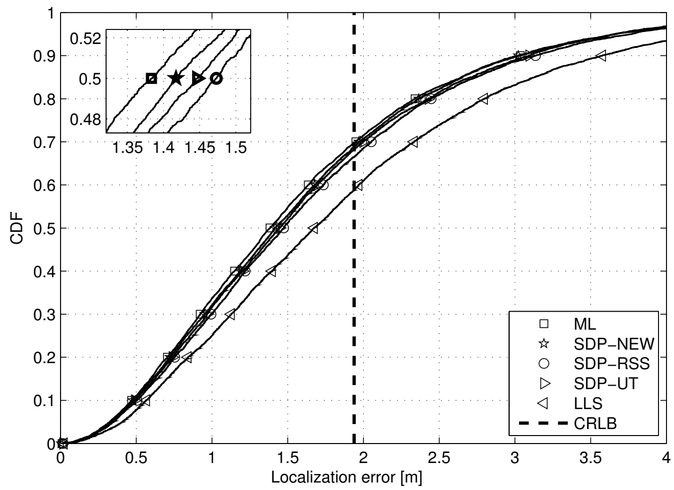

Fig. 1. The CDF of localization error for the compared estimators. $\sigma_{dB} =$ 4 dB, $\rho = 0$ (uncorrelated case). The performance of the proposed SDP estimator is very close to the optimal ML estimator.

We consider a network with four anchor nodes and one source node. The locations of anchors are fixed and 121 locations for the source node are generated uniformly in a square region of 10m $\times$ 10m. In Fig. 1, we plot the cumulative distribution function (CDF) of localization error of the compared estimators when the correlation coefficients are zero, meaning that shadowing is uncorrelated. The standard deviation of shadowing σ_{dB} was set to 4 dB. The depicted CRLB is obtained by averaging over the CRLB of each source node location. Fig. 1 shows that the ML estimator is optimal and has superior performance in comparison to the other estimators, mainly because its solver is initialized with the true values. The proposed SDP (SDP-NEW) provides excellent accuracy and its performance is very close the optimal ML estimator. SDP-RSS and SDP-UT also show good performance, although they are slightly worse than ML and SDP-NEW. We later show that the running time of the proposed SDP estimator is considerably less than the other two SDP estimators, exhibiting its major advantage in uncorrelated shadowing. The LLS has inferior performance among the compared estimators.

In Fig. 2, we compare the CDF of localization error of the considered estimators when shadowing is highly correlated. The standard deviation of shadowing σ_{dB} and correlation

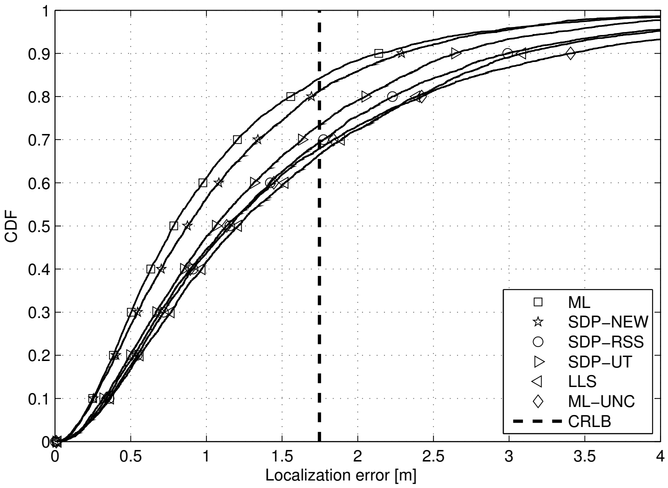

Fig. 2. The CDF of localization error for the compared estimators. $\sigma_{dB} = 4$ dB, $\rho = 0.8$ (correlated case). ML performs considerably better than ML-UNC meaning that using correlation among shadowing components can improve positioning accuracy considerably. The proposed SDP estimator is still close to ML and outperforms the other estimators.

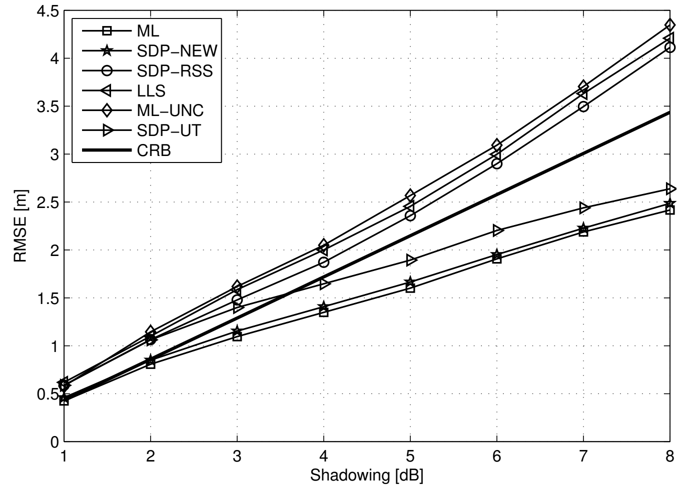

Fig. 3. The RMSE of the compared estimators versus the standard deviation of the shadowing with $\rho = 0.8$. As the shadowing increases, the difference between ML and ML-UNC increases.

coefficient were 4 dB and 0.8, respectively. The shadowing components were generated using (16). To show the effect of spatial shadowing on localization accuracy, we plot the ML estimator in two situations. Besides the ML estimator in (3) using the non-diagonal covariance matrix $\mathbf{Q}$, we also plot an ML estimator (labeled as ML-UNC) which uses a diagonal covariance matrix $\mathbf{Q} = \sigma_{dB}^2 \mathbf{I}_M$ and neglects the correlation among shadowing components. Comparing the CRLB in Fig. 1 and Fig. 2, we can see that the localization accuracy improves about 10% in correlated shadowing. However, the estimators should take the correlations into account in order to exploit this improvement. ML considering shadowing correlations shows about 0.7 m improvement at 70% CDF in comparison with the uncorrelated case in Fig. 1. However, ML-UNC neglecting shadowing correlations shows only a slight improvement in this case. The proposed SDP estimator exhibits good improvement and its performance is still very close to ML. The improvement is not dramatic for other estimators, SDP-RSS, SDP-UT, and LLS, since these estimators do not consider the correlations among shadowing components or cannot be easily adapted to do so.

In Table I, we compare the average running time of the considered estimators. We considered the same network configuration as in Fig. 2 with the standard deviation of shadowing of 4 dB and the correlation of 0.8. The linear estimator has the fastest running time, since the estimator requires simple calculations and all of them are done in one iteration. ML and ML-UNC run faster than the SDP estimators. However, it should be noted that the ML estimator is initialized with the true values which decreases the running time significantly as the solver converges after some iterations [5]. ML requires a larger number of iterations and has higher running than ML-UNC. The reason in that the cost function of ML-UNC has only quadratic terms and is less complex than the that of ML. This is the cost that must be paid for higher accuracy. As can be seen from Table I, among SDP estimators, the proposed SDP estimator has not only the highest accuracy but also the lowest running time.

Fig. 3 shows the root mean square error (RMSE) of the compared estimators versus the standard deviation of the shadowing. The same network configuration is considered. The correlation coefficient ρ was set to 0.8. The RMSE for each estimator is computed by averaging over all source node locations and random noise realizations. The difference between ML and ML-UNC increases as the standard deviation of the shadowing increases. Therefore, considering correlations among measurement in severe shadowing is more beneficial. The proposed SDP estimator provides excellent performance which is very close to the ML estimator accuracy for all shadowing range. For shadowing larger than 2 dB, the performance of ML, SDP-NEW, and SDP-UT is lower than the CRLB. The reason is that the estimators are biased and we cannot expect the CRLB to provide a lower bound on their accuracy. This phenomenon happened frequently in other studies [5], [16]. Among ML-UNC, SDP-RSS, SDP-UT, and LLS that do not consider correlated shadowing, SDP-UT provides the best accuracy.

In the previous simulations, we assumed that the exact values of the covariance matrix are known to the estimator. However, in practical case, this assumption might not be valid as the exact value of the correlation may be difficult to determine. Here, we examine the sensitivity of the proposed estimator to the knowledge of the covariance matrix. Now, we assume that the exact values of ρ_{ij} are not available to the estimator. Instead, it has an approximate (estimated) value $\hat{\rho}_{ij}$:

$$\hat{\rho}_{ij} = \rho_{ij} + q_{ij} \tag{17}$$

where q_{ij} is modeled as iid truncated Gaussian random variables with the variance of 0.01 which defines the uncer-

TABLE II
THE RMSE OF THE COMPARED ESTIMATORS WITH THE TRUE AND APPROXIMATE COVARIANCE MATRIX (CM). $\sigma_{dB} = 4$, $\rho_{ij} = 0.8$.

Estimator	True CM	Approximate CM
ML	1.35	1.48
SDP-NEW	1.41	1.53

tainty on the correlation coefficients. The reason we model the uncertainty as a truncated Gaussian random variable is that the correlations $\hat{\rho}_{ij}$ must be between 0 and 1. The same configuration as in Fig. 2 was considered. Table II shows the RMSE of the proposed estimator with the true and approximate covariance matrix. As can be seen, the performance of both ML and SDP-NEW declines slightly by using an approximate covariance matrix. However, they both still provide better accuracy than the estimators which neglect the correlation among measurements (Fig. 3). The reader is referred to [5] for the sensitivity of the proposed estimator to the knowledge of the transmit power and the path-loss exponent in uncorrelated shadowing.

VI. CONCLUSION

We examined RSS-based wireless sensor localization in spatially correlated shadowing environments. Avoiding the ML estimator convergence problem, we derived a novel SDP technique by converting the ML estimator into a convex estimator. It was shown that when RSS measurements are correlated, the estimation accuracy of the source location can be significantly improved. Computer simulations were conducted to compare the proposed SDP estimator with the ML estimator and previously considered estimators. Results showed that the performance of the proposed estimator approaches the ML estimator accuracy and outperforms the previously considered estimators with considerably lower complexity in both uncorrelated and correlated shadowing environments.

VII. RELATION TO PRIOR WORK

The current work is related to the work in [8] and [16]. The proposed SDP estimator in the current work has two advantages over the SDP estimators in [8] and [16]. First, the proposed SDP estimator considers the correlation among RSS measurement in spatially shadowing environments. Second, the proposed SDP estimator provides higher performance with less complexity.

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